A-2935
First Year B. Sc. (Sem. I) Examination
February / March – 2015
CCM-102 CS : Mathematics for Comp. Sc. - II
(\textit{Calculus - I})

Time : Hours [Total Marks : 70]

Instructions:

(1) Fill up strictly the details of signs on your answer book.

Name of the Examination : F.Y. B.Sc. (SEM. 1)
Name of the Subject : CCM-102 CS : MATHEMATICS FOR COMP. SC. - II
Subject Code No. : 2 9 3 5 Section No. (1, 2,....) : Nil

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

1 Answer the following questions : 10

(1) If \( f(x) = (1 + 3x)^{\frac{1}{x}}; x \neq 0 \) is continuous at \( x = 0 \) then
\( f(0) = \) _______.

(2) If \( y = \sin x + \cos x \) then find \( y_{4n} \).

(3) Explain ONE ONE AND ONTO functions with illustration.

(4) Evaluate \( \int_{0}^{1} \frac{2x}{x^2 + 1} \, dx \).

(5) The integral of \( \cos x \) is \( \sin x \) then the integral of \( \cos(ax + b) = \) _______.
2. (a) If \( f(x) = \frac{1}{2x+1} x \neq -\frac{1}{2} \) then prove that

\[ f(f(x)) = \frac{2x+1}{2x+3}, x \neq -\frac{3}{2}. \]

OR

(a) Show that \( f(x) = 3x^2 + 2x - 1 \) is continuous at 2.

(b) Attempt any two:

1. Define Domain and Range of the function and find the same for \( f(x) = \frac{x^2 - 9}{x - 3}. \)

2. If \( f(x) = \sqrt{x} \) and \( g(x) = \frac{1}{x^2} \) then find the domains of the \( fog \) and \( f/g. \)

3. If \( f(x) = 5x - 2; x \in \mathbb{N} \) If range of the function is \( \{3, 8, 13\} \) then find the Domain.

4. If \( f \) is a real function defined by \( f(x) = \frac{x - 1}{x + 1} \) then prove that \( f(2x) = \frac{3f(x) + 1}{f(x) + 3}. \)

3. (a) Obtain \( \lim_{\theta \to +\infty} 7\theta^3 + 8\theta^2 + 5\theta - 7. \)

OR

(a) Show that \( f(x) = \sin x \) is continuous for every value of \( x. \)
(b) Attempt any two:

(1) Examine the continuity of the function defined by

\[ f(x) = \begin{cases} 
  -x^2, & x \leq 0 \\
  5x-4, & 0 < x < 1 \\
  4x^2-3x, & 1 < x < 2 \\
  3x+4, & x \geq 2 
\end{cases} \]

at the points 0, 1, 2.

(2) Obtain \( \lim_{x \to 4} \frac{x^2-16}{\sqrt{x^2+9}-5} \)

(3) Obtain \( \lim_{x \to 3} \left( \frac{\sqrt{x+3} - \sqrt{6}}{x^2-9} \right) \)

(4) Investigate the points of continuity and discontinuity of the function \( f \) defined by

\[ f(x) = \begin{cases} 
  \frac{x^2}{a}, & x \leq a \\
  a - \frac{a^2}{x}, & x > a 
\end{cases} \]

4 (a) Find the \( n^{th} \) derivative of \( \sin(ax+b) \).

OR

(a) If \( y = \log(ax+b); a, b, x \in \mathbb{R} \) then find \( y_n \).

(b) Attempt any two:

(1) Obtain \( n^{th} \) derivative of \( \frac{x^2}{(x+2)(2x+3)} \)

(2) Obtain the \( n^{th} \) derivative of \( (ax+b)^m \).

(3) Obtain derivative of \( y = \frac{x-1}{(x-2)(x-3)} \)

(4) Obtain \( n^{th} \) derivative of \( \log \left( x + \sqrt{1+x^2} \right) \) at \( y = 0 \).
5 (a) Evaluate: \( \int \log(x+1) \, dx \).

OR

(b) Attempt any two:

1. Evaluate: \( \int \frac{dx}{\cos(x+a) \cos(x+b)} \)

2. Find the area of the region bounded by the parabola \( y^2 = 4x \) and the line \( y = 4x \).

3. Evaluate: \( \int \frac{dx}{\sin x(a+b \cos x)} \)

4. Evaluate: \( \int \frac{x \, dx}{x^4 - x^2 + 1} \).