* A - 3 0 1 4 *

A-3014
B. Sc. (Sem. III) Examination
March / April – 2015
Mathematics : Paper - MTH - 301
(Advanced Calculus - I)

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. Sc. (Sem. III)**

Name of the Subject :
☛ **Mathematics : MTH - 301 (Advanced Calculus - I)**

☛ Subject Code No. : **3 0 1 4** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) First question is compulsory.
(3) Figures to the right indicate marks of question.
(4) Follow usual notations.

1 Answer any **five** of the following questions : **10**

(1) Obtain $\lim_{x\to 0}\left\{\lim_{y\to 0}\frac{x-y}{x+y}\right\}$ and $\lim_{y\to 0}\left\{\lim_{x\to 0}\frac{x-y}{x+y}\right\}$.

(2) Obtain U_x, U_y and U_z for $U(x,y,z)=\frac{1}{3}xyz+\log\frac{yz}{x^2}$.

(3) If $x=u(1+v), y=v(1+u)$ then find $J(x,y)$.

(4) If $x=r\cos\theta, y=r\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

(5) If $f_{xx}(a,b)=r, f_{xy}(a,b)=s, f_{yy}(a,b)=t$ then write down your conclusions regarding the different values of $rt - S^2$.

(6) Define : divergence and curl of a vector point function.

(7) If $\vec{r}=(1-\cos t)\hat{i}+(t-\sin t)\hat{j}+(t^3+t^2+t+1)\hat{k}$, then find $\frac{d\vec{r}}{dt}$.

(8) If $f=x^2y+y^2x+z^2$, then find the value of ∇f at point (1, 0 –2).

2 (a) If f is a differentiable function in x and y such that $xf_x + yf_y = mf(x,y)$ then prove that f is homogeneous function of degree m. **5**

OR

(a) Let $\phi(x)$ be the function continuous at the point $(a, b) = (a, \phi(a))$ and $\lim_{(x,y)\to(a,b)} f(x,y) = l,$ then prove that $\lim_{x\to a} f(x,\phi(x)) = l.$ **5**

(b) Attempt any **two** of the following : **10**

(1) Discuss continuity of the function *f(x, y)* at the point (0,0) where $f(x,y) = \frac{\sin(x+y)}{x+y}; x+y \neq 0$ and $f(x,y) = 1;\ x+y = 0.$

(2) If $z(x+y) = x^2 + y^2$ then prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right).$

(3) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right); x+y \neq 0$ then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$

(4) If $u = f(r),\ r^2 = x^2 + y^2 + z^2$ then prove that

$$u_{xx} + u_{yy} + u_{zz} = f''(r) + \frac{2}{r} f'(r).$$

3 (a) Obtain expression of e^{ax} sin by in terms of x and y. **5**

OR

(a) Expand $f(x,y) = y^2/x^3$ into the series expansion form up to second order terms in the powers of (x – 1) and (y + 1). **5**

(b) Attempt any **two** of the following : **10**

(1) If $u^3 + v^3 = x + y,\ u^2 + v^2 = x^3 + y^3$ then show that

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{2}\frac{y^2 - x^2}{uv(u-v)}.$$

(2) If $x = \rho\cos\phi,\ y = \rho\sin\phi,\ z = z,$ then find $\frac{\partial(x,y,z)}{\partial(\rho,\phi,z)}.$

(3) Obtain expression of e^{ax+by} in the form of powers of x and y.

(4) If $u^3+v+w=x+y^2+z^2$, $u+v^3+w=x^2+y+z^2$,

$u+v+w^3=x^2+y^2+z$ then prove that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)}=\frac{1-4(xy+yz+zx)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}.$$

4 (a) Discuss about the extreme points of $u=x^3+y^3-3axy$. **5**

OR

(a) Find extreme values of $u=x^3y^2(1-x-y)$. **5**

(b) Attempt any **two** of the following : **10**

(1) Find extreme values of $f(x,y)=x^3+y^3-3x-12y+5$.

(2) Find three positive real numbers whose sum is 24 and product is maximum.

(3) Find the interior point of the triangle, such that the sum of the squares of its distances from the vertices is minimum.

(4) Show that $f(x,y)=2(x-y)^2-x^4-y^4$ is maximum at $(-\sqrt{2},\sqrt{2})$. Also find the maximum value.

5 (a) Define gradient of a scalar point function. If f and g are differentiable scalar functions, then show that $grad(fg)=f\ grad\ g+g\ grad\ f$. **5**

OR

(a) If $\vec{U}$ and $\vec{V}$ are differentiable vector functions, then prove that $curl\ (\vec{U}+\vec{V})$ = $curl\ \vec{U}+curl\ \vec{V}$. **5**

(b) Attempt any **two** of the following : **10**

(1) Define gradient of scalar point function. In usual notations prove that curl grad $f=\vec{0}$.

(2) State the condition for a vector point function to be solenoidal. If $\vec{f}=(x+y)\hat{i}+(y-z)\hat{j}(x+az)\hat{k}$ is solenoidal then find the value of a.

(3) If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t,$ where $\vec{a}$ and $\vec{b}$ are constant vectors and ω is a constant scalar, then prove that $\frac{d^2\vec{r}}{dt^2} + \omega^2\vec{r} = \vec{0}.$

(4) If $\vec{f} = (x^2yz)\hat{i} + (xy^2z)\hat{j} + (xyz^2)\hat{k},$ then find *div (curl* $\vec{f}$ *).*
