* A - 3 0 1 5 *

A-3015

Second Year B. Sc. (Sem. III) Examination

March/April – 2015

Mathematics : CCM - 302

(Ordinary Differential Equations)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.	Seat No. :
Fillup strictly the details of ☛ signs on your answer book.	
Name of the Examination :	
☛ **SECOND YEAR B. SC. (SEM. III)**	
Name of the Subject :	
☛ **MATHEMATICS : CCM - 302**	
☛ Subject Code No. : **3 0 1 5** ☛ Section No. (1, 2,.....) : **Nil**	Student's Signature

(2) All questions are compulsory.

(3) Digits to the right indicate marks of that question.

(4) Use of scientific non-programmable calculator is permissible.

1 Answer the following as directed : (any **five** out of eight) **10**

(1) Find complimentary function of $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}=0$.

(2) Find complimentary function of $\left(D^3+3D^2+3D+1\right)y=0$; $D\equiv\frac{d}{dx}$.

(3) Convert $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=2\log x$ to linear differential equation with constant coefficients.

(4) Convert $\left(x^3D^3-4x^2D^2+5xD-2\right)y=x^3$ to linear differential equation with constant coefficients ; $D\equiv\frac{d}{dx}$.

(5) Convert $(3x+2)^2\frac{d^2y}{dx^2}+5(3x+2)\frac{dy}{dx}-3y=x^2+x+1$ to linear differential equation with constant coefficients; $D\equiv\frac{d}{dx}$.

(6) Find y_1 for $(x+1)\frac{d^2y}{dx^2}-(2x+3)\frac{dy}{dx}+(x+2)y=(x+1)^2e^x$.

(7) Find y_1 such that first ordered derivative is eliminated from $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+4x^2y=xe^{x^2}$.

(8) Find y_1 such that first ordered derivative is eliminated from $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}+5y=e^x\sec x$.

2 (a) If $D\equiv\frac{d}{dx}$; $f(D)$ is a polynomial in D, with constant coefficients, of degree n then show that the particular integral of linear differential equation $f(D)y=\cos ax\,(a\in R)$ is given by 5

$$\frac{1}{\phi\left(D^2\right)}\cos ax=\frac{1}{\phi\left(-a^2\right)}\cos ax;\ \phi\left(-a^2\right)\neq 0.$$

OR

(a) If $D\equiv\frac{d}{dx}$; $f(D)$ is a polynomial in D, with constant coefficients, of degree n then show that the particular integral of linear differential equation $f(D)y=V(x)e^{ax}\,(a\in R)$ is given by

$$\frac{1}{f(D)}V(x)e^{ax}=V(x)\frac{1}{f(D+a)}e^{ax}.$$

(b) Attempt any **two** out of three : **10**

(1) Solve : $\left(D^2-(a+b)D+ab\right)y=e^{ax}+e^{bx}$; $D\equiv\frac{d}{dx}$.

(2) Solve : $\frac{d^3y}{dx^3}+y=\sin 3x+\cos^2\frac{x}{2}$.

(3) Solve : $\frac{d^3y}{dx^3}+4y=\sin 3x+e^x$.

3 (a) Describe the method of finding the general solution of **5**

$$x^n\frac{d^ny}{dx^n}+p_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}}+p_2x^{n-2}\frac{d^{n-2}y}{dx^{n-2}}+\ldots p_ny=X(x)$$

$\left(n\in N, p_1, p_2,\ldots p_n\in R\right)$.

OR

(a) Describe the method of finding the general solution of

$$(ax+b)^n\frac{d^ny}{dx^n}+p_1(ax+b)^{n-1}\frac{d^{n-1}y}{dx^{n-1}}+p_2(ax+b)^{n-2}\frac{d^{n-2}y}{dx^{n-2}}+\ldots p_ny=X(x)$$

$\left(a\neq 0, n\in N, p_1\ldots p_n\in R\right)$.

(b) Attempt any **two** out of three : **10**

(1) Solve : $x^3\frac{d^3y}{dx^3}+2x^2\frac{dy}{dx}+2y=10\left(x+\frac{1}{x}\right)$.

(2) Solve : $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=x^2\log x$.

(3) Solve : $(x+1)^2\frac{d^2y}{dx^2}-3(x+1)\frac{dy}{dx}+4y=x^2$.

4 (a) Describe the method of finding the general solution of **5**

$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=R(x)$ by method of solution in terms of known integral.

OR

(a) Describe the method of finding the general solution of

$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=R(x)$ by method of removal of the first ordered derivative.

(b) Attempt any **two** out of three : **10**

(1) Solve : $x^2 \frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$.

(2) Solve : $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} - n^2 y = 0$.

(3) Solve : $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + \left(4x^2 - 1\right)y = (-3)e^{x^2} \sin 2x$.

5 (a) Describe the method of finding the general solution of $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$ by changing independent variable x to $z = \int e^{-\int p dx} dx$. **5**

(b) Attempt any **two** out of three : **10**

(1) Solve : $x^6 \frac{d^2y}{dx^2} + 3x^5 \frac{dy}{dx} + a^2 y = \frac{1}{x^2}$.

(2) Solve : $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2 y = x^4$.

(3) Solve : $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + \left(\cos^2 x\right)y = 0$.