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A-3017
Second Year B. Sc. (Sem. III) (Computer Science) Examination
March / April – 2015
CCM-301 - CS : Advanced Calculus

Time : 3 Hours] [Total Marks : 70

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.	Seat No. :
Fillup strictly the details of ☛ signs on your answer book.	
Name of the Examination :	
☛ SECOND YEAR B. SC. (SEM. 3) (COMPUTER SCI.)	
Name of the Subject :	
☛ CCM-301 - CS : ADVANCED CALCULUS	
☛ Subject Code No. : 3 0 1 7 ☛ Section No. (1, 2,.....) : Nil	Student's Signature

(2) All questions are compulsory.

(3) Digits shown on right hand side indicate full marks of the question.

(4) Symbols have their usual meaning.

1 Attempt the questions as directed : **10**

(1) Evaluate $\int_0^1 x^4 (1-2x)^3 \, dx$

(2) Show that sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\ldots$ is convergent.

(3) If $f(x, y) = e^{xy}$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(4) Evaluate $\int_0^1 \int_0^x xy^2 dy \, dx$

(5) If $f(x, y) = \begin{cases} \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}, & (x, y) \neq (0, 0) \\ 2 \quad , & (x, y) = (0, 0) \end{cases}$

then show that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 0$.

2 (a) Show that $\int_0^1 x^{m-1}\left(1-x^2\right)^{n-1} dx = \frac{1}{2} B\left(\frac{m}{2}, n\right)$. **5**

OR

(a) Change the order of Integration of **5**

$$\int_0^a \int_{\sqrt{a^2-y^2}}^{a+y} f(x, y)\, dy\, dx$$

(b) Attempt any two : **10**

(1) Show that $\int_0^2 x^4\left(8-x^3\right)^{-1/3} dx = \frac{16}{3} B\left(\frac{5}{3}, \frac{2}{3}\right)$

(2) Evaluate $\int_0^a \int_0^{\sqrt{ay}} xy\, dy\, dx$

(3) Evaluate $\int_0^1 x^4\left(1-\sqrt{x}\right)^5 dx$

(4) Change the order of integration of

$$\int_0^{a/\sqrt{2}} \int_x^{\sqrt{a^2-x^2}} f(x, y)\, dy\, dx$$

3 (a) State and prove Euler's theorem for homogeneous functions. **5**

OR

(a) If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. **5**

(b) Attempt any **two** : **10**

(1) Divide 120 into three parts so that the sum of their products taken two at a time shall be maximum.

(2) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

(3) Expand $e^x \log(1+y)$ by using Taylor's series at $(0, 0)$.

(4) If $f(x, y) = \tan(y+ax) + (y-ax)^{3/2}$, then show that

$$\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}.$$

4 (a) Test the convergence of the series **5**

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\dots\dots\dots$$

OR

(a) Test for the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$. **5**

(b) Attempt any **two** : **10**

(1) Examine for convergence or divergence of the series $\sum_{n=1}^{\infty} \sqrt{\frac{n}{n+1}}$.

(2) Test the series $\frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \dots\dots\dots\dots + \infty$ for convergence.

(3) Test the convergence of the series

$$\frac{1}{1+2^{-1}} + \frac{2}{1+2^{-2}} + \frac{3}{1+2^{-3}} + \dots\dots\dots\dots$$

(4) Test the convergence of the series

$$\frac{1}{3} + \frac{2!}{3^2} + \frac{3!}{3^3} + \dots\dots\dots\dots + \infty$$

5 (a) Evaluate $\int_0^1 (x^2+5)\,dx$ by using definition, dividing interval in n equal parts. **5**

OR

(a) Evaluate the integral $\int_0^6 f(x)\,dx,$ where **5**

$$f(x)=\begin{cases} x^2 & , \ x<2 \\ 3x-2 & , \ x\geq 2 \end{cases}$$

By using first fundamental theorem of calculus.

(b) Attempt any **two** : **10**

(1) Verify Euler's theorem for the function

$$f(x,y)=\sin^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}$$

(2) Evaluate $\int_0^{\pi/2} \sin x\,dx$ by dividing interval $\left[0, \pi/2\right)$ in four equal parts. Using Riemann integration.

(3) Test the convergence of $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$, using p-series method.

(4) Find the extreme values of the function $f(x,y)$, where $f(x,y)=x^3y^2(12-3x-4y)$.