A-3018

Second Year B. Sc. (Sem. III) Examination

March / April – 2015

CCM-302 (CS) : Discrete Mathematics

Time : Hours] [Total Marks :

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **SECOND YEAR B. SC. (SEM. 3)**

Name of the Subject :
☛ **CCM-302 (CS) : DISCRETE MATHEMATICS**

☛ Subject Code No. : 3 0 1 8 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate full marks.

1 Answer the following questions :

(1) Define simple statement with illustration.

(2) Define Logical connectives.

(3) When do you say that, two statements are logically equivalent ?

(4) Define Tautology and Contradiction.

(5) Define inverse function with illustration.

(6) Define one to one function.

(7) Define particular solution of a Homogeneous function.

(8) Define Abelian group with illustration.

(9) Define group.

(10) Define order of a recurrence relation.

2 (a) Use truth table to prove : **5**

(i) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

OR

(a) Construct the truth table for each of the following : **5**

(i) $\sim p \vee q \vee (\sim p \wedge \sim q)$

(ii) $p \wedge (\sim q \vee q)$

(b) Attempt any **two** : **10**

(1) Prove that :

(i) $\sim (p \vee q) = (\sim p) \wedge (\sim q)$

(ii) $\sim (p \wedge q) = (\sim p) \vee (\sim q)$

(2) Show that the following pairs of prepositions are logically equivalent :

(i) $p \vee (q \wedge \sim q)$ and p

(ii) $p \wedge (\sim q \vee q)$ and p

(3) Construct the truth table for :

(i) $p \vee \sim q \Rightarrow p$

(ii) $((\sim (p \wedge q)) \vee r) \Rightarrow \sim p$

(4) Obtain the disjunctive normal form of :

(i) $p \wedge (p \Rightarrow q)$

(ii) $p \vee (\sim p \Rightarrow (q \vee (q \Rightarrow \sim r)))$

3 (a) Solve the recurrence relation **5**

$a_n + 6a_{n-1} + 12a_{n-2} + 8a_{n-3} = 0$

OR

(a) Solve the recurrence relation **5**

$a_{n+2} - 3a_{n+1} + 2a_n = 1$

(b) Attempt any two : **10**

(1) Solve the recurrence relation $a_n - 8a_{n-1} + 16a_{n-2} = 0$ with the initial conditions $a_2 = 16$ and $a_3 = 80$.

(2) Solve the recurrence relation $a_n - 7a_{n-2} + 6a_{n-3} = 0$ with the initial conditions $a_0 = 8,\ a_1 = 6$ and $a_2 = 22$.

(3) Find the particular solution of the recurrence relation $a_{n+2} - 3a_{n+1} + 2a_n = b^n$ where b is some constant.

(4) Find the particular solution of

$a_{n+2} - 4a_{n+1} + 4a_n = 3n + 2^n$.

4 (a) If $f : A \to B$ and $g : B \to C$ be two one-one and onto functions then gof is also one-one and onto function - Also $(gof)^{-1} = f^{-1}og^{-1}$. **5**

OR

(a) Consider the function $f : R \to R$ defined by **5**

$$f(x) = \begin{cases} 3x - 4 & ; x > 0 \\ -3x + 2 & ; x \le 0 \end{cases}$$

Determine : (a) $f(0), f\left(\frac{2}{3}\right), f(-2)$ and

(b) $f^{-1}(0), f^{-1}(2), f^{-1}(-7)$.

(b) Attempt any **two** : **10**

(1) Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 12 & ; x > 3 \\ 2x^2 + 3 & ; -2 < x \le 3 \\ 3x^2 - 7 & ; x \le -2 \end{cases}$$

find $f^{-1}(5)$.

(2) Show that the function $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$ are inverse of each other $\forall x \in R$.

(3) Let $f : X \to X$ where $X = \{x \in R, x \ne 0\}$ is defined by $f(x) = \frac{1}{x}$. Show that $f(x)$ is a bijection.

(4) Let $f : R^+ \to R^+$ and $g : R^+ \to R^+$ be defined by $f(x) = \sqrt{x}$ and $g(x) = 3x + 1, \forall x \in R^+$. Find out fog and gof. Is $fog = gof$?

5 (a) For an associative algebraic structure the inverse of every invertible element is unique. **5**

OR

(a) Let the binary operation * be defined as $S = \{a, b, c, d, e\}$ as shown in the following composite table. **5**

*	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	e

(i) Compute $c*d$, $b*b$, $(a*b)*c$ and $[(a*c)*e]*a$ from the composite table.

(ii) Is the operation $*$ commutative ? Justify your answer.

(b) Attempt any **two** :

(1) Prove that the fourth root of unity 1, –1, i, –i, form an abelian multiplicative group.

(2) Find the product of permutations given below. Show that it is not commutative

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$

(3) Prove that the necessary and sufficient condition for a non-empty subset H of a group $(G, *)$ to be sub group is, $a \in H,\ b \in H;\ a * b^{-1} \in H$.

(4) Show that the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 2 & 4 & 1 & 3 \end{pmatrix}$ is odd, while the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 4 & 5 & 2 & 1 \end{pmatrix}$ is even.
