AB-3152

Third Year B. Sc. (Sem. V) Examination

March / April – 2015

Mathematics : Paper - MTH-502

(Linear Algebra - I)

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **T. Y. B. Sc. (SEM. 5)**

Name of the Subject :
☛ **MATHEMATICS - MTH-502**

☛ Subject Code No. : **3 1 5 2** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate the marks of the question.
(4) Follow usual notations.

1 Answer the following : (any five) **10**

(1) In a vector space V, prove that $(-1).u = -u$, for every vector u in V.

(2) In a vector space V, prove that $-u - v = -v - u$, for all vectors u, v in V.

(3) Give an example of two distinct subspaces U and W of a vector space V such that $U \cup W$ is a subspace of V.

(4) Find the span of the set $S = \{(1,1,0),(0,0,-1)\}$ of vectors in V_3. Does $(1,1,1)$ belong to $[S]$?

(5) Are the vectors $(0,-1,1)$ and $(1,0,1)$ collinear in V_3 ? Justify your answer.

(6) Is the set $\{(1,1,0),(0,1,1),(-1,0,1)\}$ linearly independent in V_3 ?

(7) Find the co-ordinate vector of a vector $(2, 3, 4, -1)$ relative to the ordered basis $B = \{(1, 1, 0, 0), (0, 1, 1, 0), (0, 0, 1, 1), (1, 0, 0, 0)\}$ for V_4.

(8) Which is the basis for the vector space $\{\theta\}$? Find the dimension of the vector space $\{\theta\}$.

2 (a) Let R^+ be the set of all positive real numbers. The operations of addition and scalar multiplication in R^+ are defined as follows : **5**
$u + v = uv$ and
$\alpha \cdot u = u^{\alpha}$; for all u, v in R^+ and for every $\alpha \in R$.

OR

(a) If a non-empty subset S of a vector space V satisfies the conditions : **5**

(i) $u, v \in S \Rightarrow u + v \in S$;

(ii) $u \in S$ and α any scalar $\Rightarrow \alpha \cdot u \in S$, then prove that S is a subspace of V.

(b) Attempt any **two** : **10**

(1) In a vector space V, prove that $\alpha \cdot u = \theta$ if and only if either $\alpha = 0$ or $u = \theta$.

(2) Give the reason why the following subsets of V_3 are not subspaces of V_3 :

(i) $S = \{(x, y, z) \in V_3 \mid z \text{ is an integer}\}$;

(ii) $S = \{(x, y, z) \in V_3 \mid x + y + z \geq 0\}$.

(3) If u_0 is a fixed vector in a vector space V, then prove that

$[u_0] = \{\alpha \cdot u_0 \mid \alpha \text{ any Scalar}\}$ is a subspace of V.

(4) Prove that the set $S = \{(x_1, x_2, x_3) \in V_3 \mid x_1 = 0\}$ is a subspace of V_3.

3 (a) If S is a non-empty subset of a vector space V, then prove that $[S]$ is a subspace of V. **5**

OR

(a) If U and W are two subspaces of a vector space V, then prove that $U+W=[U \cup W]$. **5**

(b) Attempt any two : **10**

(1) If $v_1, v_2, \ldots\ldots, v_n$ are n vectors of a vector space V, then prove that :

(i) $[v_1, v_2, \ldots\ldots, v_n] = [\alpha_1 \cdot v_1, \alpha_2 \cdot v_2, \ldots\ldots, \alpha_n \cdot v_n]$, where each $\alpha_i \neq 0$;

(ii) $[v_1 - v_2, v_1 + v_2] = [v_1, v_2]$.

(2) If S is a non-empty subset of a vector space V, then prove that the subspace $[S]$ is the smallest subspace of V containing S.

(3) If U and W are two subspaces of a vector space V, then prove that $U \cap W$ is a subspace of V.

(4) Find the span of the set $S=\{(1,2,1),(1,1,-1),(4,5,-2)\}$ of vectors in V_3. Does $(1,0,1)$ belong to $[S]$?

4 (a) Define an LD set. Prove that any set of vectors in a vector space V containing the zero vector is LD. **5**

OR

(a) In a vector space V, suppose $\{v_1, v_2, \ldots\ldots, v_n\}$ is an ordered set of vectors with $v_1 \neq \theta$. If the set $\{v_1, v_2, \ldots\ldots, v_n\}$ is LD, then prove that $v_k \in [v_1, v_2, \ldots\ldots, v_{k-1}] : 2 \leq k \leq n$ **5**

(b) Attempt any two : **10**

(1) In a vector space V, if $v \in [v_1, v_2, \ldots\ldots, v_n]$, then prove that the set $\{v, v_1, v_2, \ldots\ldots, v_n\}$ is LD.

(2) Check whether the set $\{(1, 0, 0, 0), (1, 1, 0, 0), (0, 0, 1, 1), (1, 1, 1, 1)\}$ of vectors in V_4 is *LI* or *LD*. If *LD*, then express one of the vectors which is a linear combination of its preceding vectors.

(3) Prove that the set $\{v_1, v_2, v_3\}$ is *LD* in a vector space *V* if and only if v_1, v_2, v_3 are coplanar.

(4) Define an *LI* set. If u, v, w are *LI* vectors of a vector space *V*, then what do you say about the vectors $u-v$, $v-w$ and $w-u$? Justify your answer.

5 (a) Define a basis for a vector space. Prove that if a vector space *V* has a basis of *n* elements, then every other basis for *V* also has *n* elements. **5**

OR

(a) Prove that in an *n*-dimensional vector space *V*, any set of *n* *LI* vectors is a basis for *V*. **5**

(b) Attempt any **two** : **10**

(1) Define the dimension of a vector space. If *V* is an *n*-dimensional vector space, then prove that every set of *p* vectors; with $p > n$; is *LD*.

(2) In a vector space *V*, the set $\{v_1, v_2, ..., v_n\}$ generates *V*. If $\{v_1, v_2, ..., v_n\}$ is *LI*, then prove that the expression for any *v* in *V*; namely; $v = \alpha_1 \cdot v_1 + \alpha_2 \cdot v_2 + ... + \alpha_n \cdot v_n$, for some scalars $\alpha_1, \alpha_2, ..., \alpha_n$; is unique.

(3) Prove that the set $B = \{(1, 1, 0, 0), (0, 1, 1, 0), (0, 0, 1, 1), (1, 0, 0, 0)\}$ of vectors in V_4 is a basis for V_4.

(4) Extend an *LI* set $\{(1, 1, 1, 1), (1, 2, 1, 2)\}$ of vectors in V_4 to the basis for V_4.
