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AB-3153

B. Sc. (Mathematics) (Sem. V) Examination

March/April – 2015

Paper - MTH 503 : Real Analysis - I

(New Course)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.
Name of the Examination :
☛ **B. SC. (MATHEMATICS) (SEM. V)**
Name of the Subject :
☛ **PAPER - MTH 503 : REAL ANALYSIS - I (NEW)**
☛ Subject Code No. : **3 1 5 3** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory

(3) Digits to the right of each question indicate its marks.

(4) Follow usual symbols.

1 Answer **any five** from the following : **10**

(1) Define the upper bound of a set and find g.l.b. for the set $\left\{\pi+1, \pi+\frac{1}{2}, \pi+\frac{1}{3}, \pi+\frac{1}{4}, \ldots\ldots\right\}$

(2) Define the convergence and divergence of a sequence of a real number.

(3) If $S=\{2_n\}_{n=1}^{\infty}=\{Sn-1\}_{n=1}^{\infty}$ and $N=\{n_i\}_{i=1}^{\infty}=\{i^2\}_{i=1}^{\infty}$ then find S_5 and S_{n_3}.

(4) Classify the sequence $\left\{e^{1/n}\right\}_{n=1}^{\infty}$ into

(a) convergent (b) divergent to ∞

(c) divergent to $-\infty$ (d) oscillating.

(5) Find $N \in I$ so $\frac{1}{\sqrt{n+1}} < 0.03$ that when $n \geq N$.

(6) Define limit superior for a sequence of real numbers and find it for $\left\{\left(1+\frac{1}{n}\right)\cos n\pi\right\}_{n=1}^{\infty}$.

(7) Define a Cauchy sequence of real numbers with an illustration.

(8) Check if the sequence $\left\{\frac{1}{1+n^2}\right\}_{n=1}^{\infty}$ is monotone or not ? Justify your answer.

2 (a) If $A_1, A_2, A_3, \ldots\ldots\ldots$ are countable sets then prove that $\bigcup_{n=1}^{\infty} A_n$ is also countable. **5**

OR

(a) Show that the set of all ordered pairs of integers is countable.

(b) Answer any **two** from the following : **10**

(1) Define a countable set and prove that if B is an infinite subset of a countable set A, then B is also countable.

(2) show that if A and B are countable sets, then the Cartesian product $A \times B$ is also countable.

(3) Prove that if B is a countable subset of the uncountable set A then $A - B$ is uncountable and use it to prove that the set of all irrational numbers is uncountable.

(4) Prove that the set of all integers is countable.

3 (a) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent then prove that its limit is unique. **5**

OR

(a) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers with $S_n \geq 0$, for every n and if $\lim_{n\to\infty} s_n = L$, then prove that $L \geq 0$.

(b) Answer any **two** from the following : **10**

(1) If $L \in R, M \in R$ and $L \leq M + \varepsilon$ for all $\varepsilon > 0$, then prove that $L \leq M$.

(2) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then prove that any subsequence of $\{S_n\}_{n=1}^{\infty}$ is alos convergent to L.

(3) Let $\{S_n\}_{n=1}^{\infty}$ be the sequence defined by

$$S_1 = 1,$$
$$S_2 = 1,$$
$$S_{n+1} = S_n + S_{n-1} \quad (n = 3, 4, 5.....)$$

Then find S_8.

(4) Suppose $\{S_n\}_{n=1}^{\infty}$ converges to L then prove that $\left\{(-1)^n S_n\right\}_{n=1}^{\infty}$ converges to 0 if L = 0 and $\left\{(-1)^n S_n\right\}_{n=1}^{\infty}$ oscillates if $L \neq 0$.

4 (a) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded. **5**

OR

(a) Prove that a nonincreasing sequence which is bounded below is convergent.

(b) Answer any **two** from the following : **10**

(1) Let $s_1 = \sqrt{2}$ and let $s_{n+1} = \sqrt{2}.\sqrt{s_n}$ for $n \geq 2$ then prove that $\{S_n\}_{n=1}^{\infty}$ is nondecreasing and bounded above by $\sqrt{2}$.

(2) For $n \in I$ let $t_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!}$ then prove that $\{t_n\}_{n=1}^{\infty}$ in nondecreasing.

(3) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers, if $\lim_{n\to\infty} s_n = L$ and if $c \in R$ then prove that $\lim_{n\to\infty} cs_n = cL$.

(4) For $n \in I$ let $S_n = \frac{2 \cdot 4 \cdot 6 \cdot ...... \cdot 2n}{1 \cdot 3 \cdot 5 \cdot ......(2n-1)} . \frac{1}{n^2}$ then prove that $\{S_n\}_{n=1}^{\infty}$ in nonincreasing.

5 (a) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges, then prove that $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence. **5**

OR

(a) Let $\{S_n\}_{n=1}^{\infty}$ diverges to minus infinity and if $\{t_n\}_{n=1}^{\infty}$ is bounded then prove that $\{S_n + t_n\}_{n=1}^{\infty}$ diverges to minus infinity.

(b) Answer any **two** from the following : **10**

(1) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers, and, for each $n \in I$, let $\begin{array}{l} s_n = a_1 + a_2 + a_3 + \ldots\ldots + a_n \\ t_n = |a_1| + |a_2| + |a_3| + \ldots\ldots + |a_n| \end{array}$.

Prove the if $\{t_n\}_{n=1}^{\infty}$ is a Cauchy sequence, then so is $\{S_n\}_{n=1}^{\infty}$.

(2) If $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded.

(3) Let $\{S_n\}_{n=1}^{\infty}$ diverges to infinity and if $\{t_n\}_{n=1}^{\infty}$ converges then prove that $\{S_n + t_n\}_{n=1}^{\infty}$ diverges to infinity.

(4) If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers that diverge to infinity then prove that their sum and product also diverge to infinity.