AB-3154

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B.Sc. (Mathematics) (Sem. V) Examination

March/April – 2015

Paper - CCM - 504 : Real Analysis - II

Time : 2 Hours] [Total Marks : **50**

Instruction :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B.SC. (MATHEMATICS) (SEM. V)**

Name of the Subject :
☛ **PAPER - CCM - 504 : REAL ANALYSIS - II**

☛ Subject Code No. : **3 1 5 4** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory
(3) Digits to the right of each question indicate its marks.
(4) Follow usual symbols.

1 Answer **any five** form the following : **10**

(1) If both f and g are continuous at a then show that min. $\langle f, g \rangle$ is also continuous at a.

(2) If $0 < \delta < 1$ and if $|x-3| < \delta$ then prove that $|x^2 - x - 6| < 6\delta$.

(3) Define $\lim_{x \to a^+} f(x) = L \ in \ R^1$.

(4) Define a convergent sequence in a metric space with an illustration.

(5) Define an open ball in metric space. What is $B[a;2]$ in R_d for $a \in R_d$?

(6) Give definition of a metric for a non-empty set M.

(7) Show that every Cauchy sequence in R_d is convergent.

(8) Let $A = [0,1]$. Which of the following subsets of A are open subsets of A ?

(i) $\left[0, \frac{1}{2}\right]$ (ii) $\left(0, \frac{1}{2}\right]$

(iii) $\left[0, \frac{1}{2}\right)$ (iv) $\left(0, \frac{1}{2}\right)$

2 (a) If f and g are the real valued functions, if f is continuous at a, and if g is continuous at $f(a)$ then prove that $g \circ f$ is continuous at a. **5**

OR

(a) If f and g are the real functions with $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$. **4**

Then prove that $\lim_{x \to a} [f(x) + g(x)] = L + M$.

(b) Answer any two from the following : **10**

(1) If f is continuous at $a \in R$, then prove that $|f|$ is also continuous at $a \in R$.

(2) Prove that if limit of a function exists then it is unique.

(3) Show that if ρ and σ are both metrics for a set M then so is 2ρ and $\rho + \sigma$.

(4) For $x, y \in R$, define $\sigma(x, y) = |x - y|$. Show that σ is a metric for the set of real numbers.

3 (a) Let $\langle M, p \rangle$ be a metric space and let a be point in M. Let f and g be real valued functions whose domains are subsets of M. If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = N$. Then prove that $\lim_{x \to a} [f(x) - g(x)] = L - N$. **5**

OR

(a) Let $\langle M, p \rangle$ be a metric space and if $\{S_n\}_{n=1}^{\infty}$ is a convergent sequence of points of M then prove that $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence. **5**

(b) Answer any two from the following. **10**

(1) Define equivalent metrics and show that ρ and σ are equivalent; where ρ is usual metric on R^2 and σ is defined by $\sigma(P,Q) = |x_1 - x_2| + |y_1 - y_2|$, for $P\langle x_1, y_1\rangle$ and $Q\langle x_2, y_2\rangle$ on R^2.

(2) If σ and ρ are metrics for M and if there exists $k > 1$ such that $\frac{1}{k}\sigma(x,y) \le \rho(x,y) \le k\sigma(x,y)\ ;\ \forall x, y \in M$.

Then prove that σ and ρ are equivalent.

(3) Show that a sequence of points in any metric space cannot converge to two distinct limits.

(4) Show that if $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence in R_d then there exists $N \in I$ such that $x_N = x_{N+1} = x_{N+2} = \ldots\ldots\ldots\ldots$

4 (a) Prove that real function f is continuous at $a \in R$ if and only if $\lim_{n\to\infty} x_n = a \Rightarrow \lim_{n\to\infty} f(x_n) = f(a)$. **5**

OR

(a) If f and g are continuous functions from a metric space M_1 into a metric space M_2 then prove that $f \pm g, f.g$ are also continuous on M_1. **5**

(b) Answer any two from the following : **10**

(1) Prove that the function f is continuous at $a \in R^1$ if and only if for given $\varepsilon > 0$ $B[f(a)\ ;\ \varepsilon]$ about $f(a)$ contains an open ball $b[a;\delta]$ about a.

(2) Define metric space R_d and prove that every function from R_d into a metric space is continous on R_d.

(3) Let M be a metric space and suppose $f : M \to R_d$. Show that if f is continuous at $a \in M$ then {a} is an open ball in M.

(4) Let $\langle M_1, p_1 \rangle, \langle M_2, p_2 \rangle, \langle M_3, p_3 \rangle$ be metric spaces and let $f : M_1 \to M_2$ and $g : M_2 \to M_3$. If f is continuous at $a \in M$ and g is continuous at $f(a) \in M_2$ then prove that $g \circ f$ is continuous at a.

5 (a) Prove that every open ball in a metric space is an open set. **5**

OR

(a) If G_1 and G_2 are open subsets of the metric space M, then prove that $G_1 \cap G_2$ is also open.

(b) Answer any two from the following : **10**

(1) If A and B are open subsets of R^1 then prove that $A \times B$ is an open subset in R^2.

(2) Let f and g are continuous real valued function on the metric space M. Let $A = \{x \in M \;/\; f(x) < g(x)\}$ then prove that A is open.

(3) Let $\langle M_1, p_1 \rangle$ and $\langle M_2, p_2 \rangle$ be two metric spaces. Let $f : M_1 \to M_2$ then prove that f is continuous on M_1 if and only if whenever G is open in M_2 then $f^{-1}(G)$ is open in M_1.

(4) Lef F be any nonempty family of open subsets of a metric space M, then prove that $\bigcap_{G \in F} G$ is also an open subset of M.
