AB-3155

Third Year B. Sc. (Sem. V) Examination

March/April – 2015

Mathematics : MTH-505

(Graph Theory)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fillup strictly the details of ☛ signs on your answer book.	Seat No. :
Name of the Examination : ☛ **Third Year B. Sc. (Sem. V)**	
Name of the Subject : ☛ **Mathematics : MTH-505** *(Graph Theory)*	
☛ Subject Code No. : 3 1 5 5 ☛ Section No. (1, 2,.....) : Nil	Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding question

1 Answer the following (**any five**) : **10**

(1) Define: Selfloop, Regular graph.

(2) Find $\min l_{max}$ and $\max l_{max}$ for binary tree with 13 vertices.

(3) Construct complete graphs with four and five vertices.

(4) Draw star of David and Mohammed's scimitar.

(5) Draw Euler graph that is not arbitrarily traceable.

(6) Show that the radius of a tree is not necessarily half its diameter.

(7) Define: Walk, Path.

(8) Explain: Decomposition of a graph and complement of a subgraph in a graph.

2 (a) State and solve Konigsberg Bridge problem. **5**

OR

(a) Explain : Utility problem for three houses and three utilities.

(b) Answer the following (any two) : **10**

(1) Define: Edges in series, Null graph, infinite graph, Isolated vertex, Pendent vertex.

(2) Define simple graph. Show that the maximum number of edges in a simple graph with n vertices is n(n-1)/2.

(3) Prove that the number of vertices of odd degree in a graph is always even.

(4) Show that sum of the degrees of all vertices in a graph G is twice the number of edges in G.

3 (a) Explain: Union of graphs and Ring sum of graphs. **5**

OR

(a) Explain: Isomorphism of two graphs.

(b) Answer the following (any **two**) : **10**

(1) Explain : Path, Circuit .

(2) Prove that if a graph G has exactly two vertices of odd degree then there must be a path joining these two vertices.

(3) Explain subgraph, complement of subgraph in a graph and decomposition of a graph with illustration.

(4) State necessary conditions for isomorphic graphs. Show that these conditions are not sufficient.

4 (a) Show that a simple graph with n vertices and k components can have at most (n-k) (n-k+1)/2 edges. **5**

OR

(a) Prove that a given connected graph G is an Euler graph iff all vertices of G are of even degree.

(b) Answer the following (any **two**) : **10**

(1) An Euler graph G is arbitrarily traceable from vertex v in G if and only if every circuit in G contains v.

(2) Prove that a simple graph with n vertices must be connected if it has more than [(n-1)(n-2)]/2 edges.

(3) Show that in a complete graph with n vertices there are (n-1)/2 edge disjoint Hamiltonian circuits if n is odd number and $n \geq 3$.

(4) Explain : Hamiltonion circuit, Unicursal graph.

5 (a) Show that any connected graph with n vertices and n-1 edges is a tree. **5**

OR

(a) Define tree. Show that if in a graph G there is one and only one path between every pair of vertices then G is a tree.

(b) Answer the following **(any two)** : **10**

(1) Show that a graph with n vertices, n-1 edges and no circuit is connected.

(2) Explain: Distance between two trees, Eccentricity of a vertex and centre of a graph.

(3) Show that every tree has either one or two centers.

(4) Explain: Rooted tree and Binary tree.
Show that the number of vertices n in a binary tree is always odd.