* A B - 3 1 5 6 *

AB-3156

B. Sc. (Sem. V) Examination

March/April – 2015

Mathematics : Paper - 506

(Number Theory - I)

Time : **3** Hours] [Total Marks : **70**

Instruction :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. SC. (SEM. V)**

Name of the Subject :

☛ **MATHEMATICS : PAPER - 506**

☛ Subject Code No. : **3 1 5 6** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All the questions are compulsory.

(3) Figures to the right indicate full marks of the question.

(4) Follow usual notations.

1 answer any five questions : **10**

(1) Prove that if $a|b$ and $b \neq 0$ then $|a| \leq |b|$.

(2) For some integers x and y if $ax + by = 1$, then prove that $gcd\ (a,b) = 1$.

(3) If a is a positive integer then show that $a^2 + a + 1$ is not a perfect square.

(4) When two integers, a and b are said to be relatively prime ?

(5) Check whether 403 is a prime or composite ?

(6) Find the remainder when $20^{20} - 1$ is divided by 41.

(7) For any integer a, prove that $a^4 \equiv 0$ or 1 (mod 5).

(8) If $N = 3417x$ is divisible by 11, then find x.

2 (a) Given integers a and b not both zero, prove that $gcd(a,b)$ exists. **5**

OR

(a) For positive integers a and b prove that $gcd(a, b). lcm(a, b) = ab$. **5**

(b) Attempt any two. **10**

(1) Prove that cube of any integer has one of the form : $9k, 9k+1$, or $9k+8$.

(2) Find integers x and y such that $\gcd(24, 138) = 24x + 138y$.

(3) For non zero integers a and b, prove that $\gcd(a,b) = \text{lcm}(a,b)$ if and only if $a = \pm b$.

(4) If a and b are odd integers then prove that $32 \mid (a^2+3)(a^2+7)$.

3 (a) Show that the linear Diophantine equation $ax + by = c$ has a solutin if and only if $d \mid c$, where $d = gcd(a,b)$. **5**

OR

(a) If p is a prime and $p \mid a_1 a_2 \ldots a_n$, then prove that $p \mid a_k$ for some k, where $1 \le k \le n$.

(b) Attempt any two : **10**

(1) Determine all solutions in the integers of the Diophantine equeation $56x + 72y = 40$.

(2) Determine all solutions in the positive integers of the Diophantine equation $54x + 21y = 906$

(3) Prove that a prime of the form $3n+1$ is also of the form $6m+1$.

(4) If $n > 4$ is composite then prove that $n \mid (n-1)!$.

4 (a) Prove or disprove : There are finite numbers of primes. **5**

OR

(a) For $n > 1$ show that there are at least $n + 1$ primes less that 2^{2^n}.

(b) Attempt **any two** : **10**

(1) If $a_1, a_2, ..., a_n$ is a complete set of residues modulo n, and $\gcd(2, n) = 1$, then prove that $2a_1, 2a_2, ..., 2a_n$ is also a complete set of residues modulo n.

(2) If $\gcd(a, n) = 1$, then show that the integers $c, c + a, c + 2a, c + 3a, ..., c + (n-1)a$ form a complete set of residues modulo n, for any integer c.

(3) Prove that the congruence relation modulo n, of integers, is an equivalence relation.

(4) Prove that the product of any set of n consecutive integers is divisible by n.

5 (a) Let $P(x) = \sum_{k=0}^{m} c_k x^k$ be a polynomial function of x with integral coefficients c_k. If $a \equiv b \pmod{n}$, then prove that $P(a) \equiv P(b) \pmod{n}$. **5**

OR

(a) If $a \equiv b \pmod{n}$ then prove that $\gcd(a, n) = \gcd(b, n)$.

(b) Attempt **(any two)** : **10**

(1) Find the remainder obtained when the sum $1! + 2! + ... + 99! + 100!$ is divided by 15.

(2) Prove that the integer $53^{103} + 103^{53}$ is divisible by 39.

(3) Prove that 16^{53} is divisible by 7.

(4) Working modulo 9 or 11, find the missiong digit, x in the following calculation : $2x99561 = [3(523 + x)]^2$.
