AC-2968
First Year B. Sc. (Comp. Sci.) (Sem. II) Examination
April / May - 2015
Paper - III : Mathematics for Computer Science - III
(Discrete Mathematics - II)

Time : 3 Hours] [Total Marks : 70

Instructions :
(1) Fill up strictly the details of signs on your answer book.

(2) All questions are compulsory.
(3) Figures to the right indicate full marks.
(4) Symbols have their usual meaning.

1 Attempt the following as directed : 10

(1) Represent the following sets in roster or tabular form

(a) \( A = \{ x \mid x \text{ is an even prime} \} \)

(b) \( A = \{ x \mid x \in \mathbb{R} \text{ and } x^2 - 1 = 0 \} \)

(2) Define a relation \( R = \{(1, 2), (2, 2), (2, 3)\} \) on the set \( A = \{1, 2, 3\} \). Determine whether \( R \) is Antisymmetric.

(3) Define bounded lattice.

(4) Complement the following expression by applying De-Morgan’s rule.

\[ (A + A'B)' \]

(5) Suppose \( A = \{3, 4, 5, 6\} \), \( B = \{ x \in \mathbb{Z} \mid x \text{ is even} \} \) find \( B - A \).
2 (a) If \( X = \{a, b, c\}, \ Y = \{d, e\}, \ Z = \{a, b, c, d, e\} \) find

(i) \( X \times Y \)  
(ii) \( Z \times Y \)  
(iii) \( Y \times Y \) and hence prove that

\((Z \times Y) - (X \times Y) = Y \times Y.\)

OR

(a) Prove that \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)

(b) Attempt any two:

(i) If \( A, B, C \) are three sets such that \( A \subseteq B \), show that

\( (A \times C) \subseteq (B \times C). \)

(ii) If \( A = \{1, 2, 3\}, \ B = \{2, 3, 4\} \) and \( U = \{1, 2, 3, 4, 5, 6\} \)

verify that (i) \( (A \cup B)' = A' \cap B' \) (ii) \( (A \cap B)' = A \cup B' \),

where ‘\( ' \) denotes the complement of the respective set.

(iii) If \( A_i = [i, i+1] \), where \( i \in Z \) the set of integers, find

(i) \( A_2 \cup A_3 \) (ii) \( A_3 \cup A_4. \)

(iv) Out of 80 students in a class, 60 play football, 53 play hockey and 35 both the games. Find how many students do not play of these games.

3 (a) If \( R \) be a relation in the sets of integers \( z \) defined by

\( R = \{ (x, y) | x \in Z, y \in Z, (x - y) \text{ is divisible by } 6 \} \).

Then prove that \( R \) is an equivalence relation.

OR

(a) Let \( A = \{1, 2, 3, 6\} \), if for \( x, y \in A \)

\( R = \{ (x, y) | x \leq y \} \)

\( S = \{ (x, y) | x \text{ divides } y \} \)

Write \( R \) and \( S \) as sets in roster form and find \( R \cap S. \)

(b) Attempt any two:

(i) Let \( A = \{2, 4\} \) and \( B = \{6, 8, 10\} \) and define binary relation \( R \) and \( S \) from \( A \) to \( B \) as follows

\( \forall (x, y) \in A \times B, \ xRy \iff x \text{ divides } y \)

\( \forall (x, y) \in A \times B, \ xSy \iff y - 4 = x \)

state explicitly which ordered pairs are in

\( A \times B, R, S, R \cup S \) and \( R \cap S. \)

AC-2968] 2 [ Contd......
(ii) The relation $R$ on the set $\{1, 2, 3, 4, 5\}$ defined by the rule, $(x, y) \in R$ if $x + y \leq 6$, find

(a) elements of $R$
(b) elements of $R^{-1}$
(c) Domain of $R$
(d) Range of $R$
(e) Domain of $R^{-1}$

where $R^{-1}$ is the inverse of $R$.

(iii) Show that the relation 'less than or equal to' on the set of integers is a partial order.

(iv) Show that the relation $R$ defined as $R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} | x - y \leq 0\}$ is a partial order relation.

4 (a) Show that the set $B = \{0, 1\}$ together with the operations $+$ and $\cdot$ defined on $B$ as follows:

<table>
<thead>
<tr>
<th>+</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\cdot$</th>
<th>0</th>
<th>1</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

$0' = 1$ and $1' = 0$ is a Boolean algebra.

OR

(a) Define distribution lattice and show that $L = (P(S), \cup, \cap)$, where $P(S)$ is a power set of set is a distributive lattice.

(b) Attempt any two:

(1) Show that the following diagram defines lattice.

```
   a
  / \  \
 b   d
   \ /  \
    c
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(2) State and prove that the absorption rule in Boolean algebra.

(3) Find the value of Boolean expression.
   \[ f(1, 0, 1), \text{ where} \]
   \[ f(x_1, x_2, x_3) = x_1 x_2' + x_2 x_3' + x_3 x_1' \]

(4) In a distributive lattice. Prove that if an element has complement, then this complement is unique.

5

(a) Simplify Boolean expression \( xy + xy' + x'y \).

OR

(a) If \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \) then find the set specified by the following bit strings.
   (i) 011001000
   (ii) 00001111
   (iii) 111100010
   (iv) 010111001
   (v) 110001001

(b) Solve any two:

   (1) Suppose \( (L, \wedge, \vee) \) is a complemented distributive lattice, then show that
   \( (x \vee y)' = x' \wedge y' \), \( \forall x, y \in L \)

   (2) Let \( A = \{1, 2, 3\} \), \( B = \{2, 3, 4\} \), \( C = \{5, 6, 7\} \) prove that
   \( (A - B) - C = A - (B \cup C) = (A - C) - (B - C) \)

   (iii) If \( A = \{2, 3\} \), \( B = \{-1, 2\} \), \( C = \{a, b\} \), verify that
   \[ A \times (B \cup C) = (A \times B) \cup (A \times C). \]

   (iv) Find the values of \( \alpha(1.1.0) \) and \( \alpha(1.1.1) \), where \( \alpha \) is the Boolean expression \( \alpha(x_1, x_2, x_3) = x_1 x_2' + x_1 + x_3'. \)