AC-2968

First Year B. Sc. (Comp. Sci.) (Sem. II) Examination

April / May – 2015

Paper - III : Mathematics for Computer Science - III

(Discrete Mathematics - II)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **FIRST YEAR B. SC. (COMP. SCI.) (SEM. 2)**

Name of the Subject :

☛ **PAPER - 3 : MATHEMATICS FOR COMP. SCI. - 3**

☛ Subject Code No. : **2 9 6 8** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

(4) Symbols have their usual meaning.

1 Attempt the following as directed : **10**

(1) Represent the following sets in roaster or tabular form

(a) $A = \{x \mid x$ is an even prime$\}$

(b) $A = \{x \mid x \in R$ and $x^2 - 1 = 0\}$

(2) Define a relation $R = \{(1,2),(2,2),(2,3)\}$ on the set $A = \{1,2,3\}$. Determine whether R is Antisymmetric.

(3) Define bounded lattice.

(4) Complement the following expression by applying De-Morgan's rule.

$(A + A'B)'$

(5) Suppose $A = \{3,4,5,6\}$, $B = \{x \in z \mid x \text{ is even}\}$ find $B - A$.

2 (a) If $X=\{a,b,c\}$, $Y=\{d,e\}$, $Z=\{a,b,c,d.,e\}$ find **5**
(i) $X\times Y$ (ii) $Z\times Y$ (iii) $Y\times Y$ and hence prove that $(Z\times Y)-(X\times Y)=Y\times Y$.

OR

(a) Prove that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ **5**

(b) Attempt any **two** : **10**

(i) If A, B, C are three sets such that $A\subseteq B$, show that $(A\times C)\subseteq(B\times C)$.

(ii) If $A=\{1,2,3\}$, $B=\{2,3,4\}$ and $U=\{1,2,3,4,5,6\}$ verify that (i) $(A\cup B)'=A'\cap B'$ (ii) $(A\cap B)'=A'\cup B'$, where '′' denotes the complement of the respective set.

(iii) If $A_i=[i,i+1]$, where $i\in Z$ the set of integers, find (i) $A_2\cup A_3$ (ii) $A_3\cup A_4$.

(iv) Out of 80 students in a class, 60 play foot ball, 53 play hockey and 35 both the games. Find how many students do not play of these games.

3 (a) If R be a relation in the sets of integers z defined by $R=\{(x,y)\mid x\in Z, y\in Z, (x-y)$ is divisible by 6$\}$. Then prove that R is an equivalence relation. **5**

OR

(a) Let $A=\{1,2,3,6\}$, if for $x,y\in A$ **5**

$R=\{(x,y)\mid x\le y\}$

$S=\{(x,y)\mid x \text{ divides } y\}$

Write R and S as sets in roaster form and find $R\cap S$.

(b) Attempt any **two** : **10**

(i) Let $A=\{2,4\}$ and $B=\{6,8,10\}$ and define binary relation R and S from A to B as follows

$\forall(x,y)\in A\times B,\ xRy\Leftrightarrow x \text{ divides } y$

$\forall(x,y)\in A\times B,\ xSy\Leftrightarrow y-4=x$

state explicitly which ordered pairs are in $A\times B, R, S, R\cup S$ and $R\cap S$.

(ii) The relation R on the set $\{1, 2, 3, 4, 5\}$ defined by the rule. $(x, y) \in R$ if $x + y \leq 6$, find

(a) elements of R

(b) elements of R^{-1}

(c) Domain of R

(d) Range of R

(e) Domain of R^{-1}

where R^{-1} is the inverse of R.

(iii) Show that the relation 'less than or equal to" on the set of integers is a partial order.

(iv) Show that the relation R defined as $R = \{(x, y) \in z \times z \mid x - y \leq 0\}$ is a partial order relation.

4 (a) Show that the set $B = \{0, 1\}$ together with the operations + and • defined on B as follows : **5**

+	0	1
0	0	1
1	1	0

•	0	1
0	0	1
1	0	1

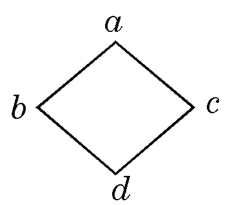
$0' = 1$ and $1' = 0$ is a Boolean algebra.

OR

(a) Define distribution lattice and show that $L = (P(S), \cup, \cap)$, where $P(S)$ is a power set of set is a distributive lattice. **5**

(b) Attempt any **two** : **10**

(1) Show that the following diagram defines lattice.



(2) State and prove that the absorption rule in Boolean algebra.

(3) Find the value of Boolean expression.

$f(1, 0, 1)$, where

$f(x_1, x_2, x_3) = x_1 x_2' + x_2 x_3' + x_3 x_1'$

(4) In a distributive lattice. Prove that if an element has complement, then this compliment is unique.

5 (a) Simplify Boolean expression $xy + xy' + x'y$. **5**

OR

(a) If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ then find the set specified by the following bit strings. **5**

(i) 011001000 (ii) 000011111

(iii) 111100010 (iv) 010111001

(v) 110001001

(b) Solve any two : **10**

(1) Suppose $(L, \wedge, \vee)$ is a complemented distributive lattice, then show that

$(x \vee y)' = x' \wedge y', \ \forall\, x, y \in L$

(2) Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{5, 6, 7\}$ prove that

$(A - B) - C = A - (B \cup C) = (A - C) - (B - C)$

(iii) If $A = \{2, 3\}$, $B = \{-1, 2\}$, $C = \{a, b\}$, verify that

$A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(iv) Find the values of $\alpha(1, 1, 0)$ and $\alpha(1, 1, 1)$, where α is the Boolean expression $\alpha(x_1, x_2, x_3) = x_1 x_2' + x_1 + x_3'$.