AC-3099

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B. Sc. (Sem. IV) Examination

March/April – 2015

Mathematics : MTH - 401

(Advanced Calculus - II)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. SC. (SEM. IV)**

Name of the Subject :
☛ **MATHEMATICS : MTH - 401**

☛ Subject Code No. : **3 0 9 9** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) First question is **compulsory**.
(3) Figures to the right indicate marks of corresponding question.
(4) Follow usual notations.

1 Answer **any five** of the following questions : **10**

(1) Evaluate : $\int_1^2 \int_2^3 \left(y^2+2xy\right)dy\,dx$.

(2) Find Laplace transform of the function $F(t)=e^{at}$.

(3) Evaluate : $\int_0^1 x^4(1-x)^3\,dx$.

(4) Show that $\beta(m,n)=\beta(m+1,n)+\beta(m,n+1)$.

(5) Check the validity of $\int_0^2 \int_2^3 dx\,dy = \int_0^2 \int_2^3 dy\,dx$.

(6) Evaluate : $L^{-1}\left[\dfrac{1}{p^2-6p+10}\right]$.

(7) Find $L\left\{3t^4-2t^2+4e^{-3t}+3\cosh 2t\right\}$.

(8) Evaluate : $L^{-1}\left[\dfrac{1}{(p+a)^n}\right]$ for $n \in N$.

2 (a) Find the volume of the tetrahedron enclosed by the coordinate planes and the plane $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$. **5**

OR

(a) Change the order of integration of the double integral **5**

$$\int_0^4 \int_0^{\frac{y}{2}} f(x,y)\,dy\,dx.$$

2 (b) Attempt any **two** of the following : **10**

(1) Find the volume of the sphere $x^2+y^2+z^2=a^2$.

(2) Evaluate : $\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \sin(x+y)\,dy dx$.

(3) Evaluate : $\int_1^2 \int_0^y \frac{dx\,dy}{x^2+y^2}$.

(4) Change the order of integration of the double integral

$$\int_0^{2t} \int_{\sqrt{2tx-x^2}}^{\sqrt{2tx}} f(x,y)\,dxdy.$$

3 (a) State and prove the relation between Beta and Gamma functions. **5**

OR

(a) Prove that $\Gamma n = \frac{1}{n}\int_0^{\infty} e^{-y^{\frac{1}{n}}}\,dy$. **5**

3 (b) Attempt any **two** of the following : **10**

(1) Show that (i) $\lceil n+1 = n\lceil n$ (ii) $\lceil n+1 = n!$

(2) Prove that $B(l,m) = \int_0^\infty \frac{y^{l-1}}{(1+y)^{l+m}} dy = \int_0^\infty \frac{y^{m-1}}{(1+y)^{l+m}} dy$.

(3) Show that $\int_0^2 x(6-x^3)^{\frac{1}{3}} = \frac{16\pi}{9\sqrt{3}}$.

(4) Show that

$$\int_0^\infty x^{m-1} \cos bx \, dx = \frac{\lceil m}{b^m} \cos\left(\frac{m\pi}{2}\right)$$ and

$$\int_0^\infty x^{m-1} \sin bx \, dx = \frac{\lceil m}{b^m} \sin\left(\frac{m\pi}{2}\right).$$

4 (a) If $L\{F(t)\} = f(p)$ then show that $L\left[e^{at} F(t)\right] = f(p-a)$; **5**

where $p > a$. Using this property find $L\left[e^{at} \sin 3t\right]$.

OR

(a) State and prove the second shifting theorem for Laplace transform. **5**

4 (b) Attempt any **two** of the following : **10**

(1) Find Laplace transform of $F(t) = \begin{cases} \sin\left(t - \frac{2}{3}\pi\right), & t > \frac{2}{3}\pi \\ 0 & t < \frac{2}{3}\pi \end{cases}$

(2) Evaluate : $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$.

(3) Find the value of $L\{\sin\sqrt{t}\}$.

(4) Prove that $L\{t^n\} = \frac{n!}{p^{n+1}}$, where $p > 0$, $n \in N$.

5 (a) State and prove the linearity property of inverse Laplace transforms. **5**

OR

(a) In usual notation prove that

$$L^{-1}[f(p)] = F(t) \Rightarrow L^{-1}[f(ap)] = \frac{1}{a} F\left(\frac{t}{a}\right).$$

5 (b) Attempt any **two** of the following :

(1) Show that $L^{-1}\left[\frac{3p-5}{p^2-6p+25}\right] = e^{3t}(3\cos 4t + \sin 4t)$.

(2) Find $L^{-1}\left\{\frac{8-6p}{16p^2+9} - \frac{3+4p}{9p^2-16}\right\}$.

(3) Prove that if

$$L^{-1}\left[\frac{p}{(p^2+1)^2}\right] = \left(\frac{t}{2}\right)\sin t, \text{ then } L^{-1}\left[\frac{50p}{(25p^2+1)^2}\right] = \left(\frac{t}{5}\right)\sin\left(\frac{t}{5}\right).$$

(4) Show that $L^{-1}\left[\frac{1}{p}\cos\frac{1}{p}\right] = 1 - \frac{t^2}{(2!)^2} + \frac{t^4}{(4!)^2} - \frac{t^6}{(6!)^6} + \ldots\ldots$