AC-3101
B. Sc. (Sem. IV) Examination
April / May – 2015
Mathematics - MTH - 403
(Numerical Analysis - II)

Time : Hours] [Total Marks : 70
Instructions :
1. Fill up strictly the details of signs on your answer book.
Name of the Examination : B. SC. (SEM. IV)
Name of the Subject : MATHEMATICAL - MTH - 403
Subject Code No. : 3 1 0 1 Section No. (1, 2,.....) Nil

2. Answer all questions.
3. Figures to the right indicate full marks of the question.
4. Follow usual notations.
5. Use of Scientific non-programmable calculator is allowed.

Que. 1 (a) Answer any FIVE as directed : [10]

1. Use Lagrange’s Interpolation formula to obtain the function \( f(x) \) for the given data \((0, -1)\) and \((3, 2)\).
2. If \( f(x) = \frac{1}{x^2} \), then find the divided difference \([x_0, x_1, x_2]\).
3. If \( b - a = c - b = h, h > 0 \), then find \([a, b, c]\).
4. Prove : \([x_0, x_1] = [x_1, x_0]\).
5. Construct divided difference table:

\[
\begin{array}{c|c|c|c|c}
  x & -2 & -1 & 2 & 3 \\
  y(x) & -12 & -8 & 3 & 5 \\
\end{array}
\]
6. What is the necessary condition for applying the Simpson’s Rule? Why?
7. Write the formula to obtain the first derivative at \( x = x_n \) and second derivative at \( x = x_0 \).
8. Define the Initial Value Problem.

Que. 2 (a) Derive Newton’s Divided Difference Interpolation formula. [05]

OR

(a) Derive Lagrange’s Interpolation formula. [05]

AC-3101] 1 [Contd...
Que.2 (b) Attempt any TWO :

(1) Applying Lagrange’s Interpolation Formula, find a cubic polynomial which approximates the following data:

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y(x)</td>
<td>-12</td>
<td>-8</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Express the rational function \( f(x) = \frac{3x^2 + x + 1}{(x-1)(x-2)(x-3)} \) as a sum of partial fraction.

(2) Consider the following data, use Newton’s Divided Difference Interpolation formula to obtain \( \log_{10} 301 \):

<table>
<thead>
<tr>
<th>x</th>
<th>300</th>
<th>304</th>
<th>305</th>
<th>307</th>
</tr>
</thead>
<tbody>
<tr>
<td>\log_{10} x</td>
<td>2.4771</td>
<td>2.4829</td>
<td>2.4843</td>
<td>2.4871</td>
</tr>
</tbody>
</table>

(4) Using the following table, find \( f(x) \) as a polynomial in \( x \) using Newton’s Divided Difference interpolation formula :

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>3</td>
<td>-6</td>
<td>39</td>
<td>822</td>
<td>1611</td>
</tr>
</tbody>
</table>

Que.3 (a) In usual notations prove that,

\[
\left[ \frac{d^2 y}{dx^2} \right]_{x=x_0} = \frac{1}{h^2} \left[ \nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \frac{5}{6} \nabla^5 y_n + \ldots \right]
\]

OR

(a) In usual notation prove that,

\[
\left[ \frac{dy}{dx} \right]_{x=x_0} = \frac{1}{h} \left[ \Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \ldots \right]
\]

Que.3 (b) Attempt any TWO :

(1) The distances travelled by a train at different times are as given below:

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>0</td>
<td>3</td>
<td>7</td>
<td>15</td>
<td>38</td>
<td>50</td>
</tr>
</tbody>
</table>

Estimate train’s velocity for \( t = 0 \).

(2) The function \( y = f(x) \) is defined as follows:

<table>
<thead>
<tr>
<th>x</th>
<th>1.00</th>
<th>1.05</th>
<th>1.10</th>
<th>1.15</th>
<th>1.20</th>
<th>1.25</th>
<th>1.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>1.00</td>
<td>1.025</td>
<td>1.049</td>
<td>1.072</td>
<td>1.095</td>
<td>1.118</td>
<td>1.140</td>
</tr>
</tbody>
</table>

Compute the values of second derivative at \( x = 1.05 \)

(3) From the following table of values of \( x \) and \( y \), obtain the value of the first derivative when \( x = 2.2 \):

<table>
<thead>
<tr>
<th>x</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
<th>2.2</th>
</tr>
</thead>
</table>

(4) The following table of values \( x \) and \( y \) is given, find \( \frac{d^3 y}{dx^3} \) when \( x = 6 \):

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

Que.4 (a) State and prove Simpson’s Rule.

OR

AC-3101] 2 [Contd...
(a) State and prove Simpson’s $\frac{3}{8}$ Rule.  

**Que.4** (b) Attempt any TWO:

1. Find the value of $\int_3^7 x^2 \log x \, dx$ by using Trapezoidal Rule by taking 4 subintervals of $[3, 7]$.

2. Evaluate $\int_0^\pi \sqrt{\sin \theta} \, d\theta$ using Simpson’s $\frac{1}{3}$ rule with $h = \frac{\pi}{12}$.

3. Estimate the value of the integral $\int_1^3 \frac{1}{x} \, dx$ by Simpson’s rule with 8 strips.

4. Evaluate the integral $\int_0^1 \sqrt{1 - x^2} \, dx$ by taking $h = \frac{1}{6}$.

**Que.5** (a) Explain Euler’s method to solve the initial value problem $\frac{dy}{dx} = f(x, y)$, where $y(x_0) = y_0$.

OR

(a) Explain Picard’s method to solve the initial value problem $\frac{dy}{dx} = f(x, y)$, where $y(x_0) = y_0$.

**Que.5** (b) Attempt any TWO:

1. If $\frac{dy}{dx} = \frac{1}{x^2 + y}$, $y(4) = 4$, compute the value of $y(4.1), y(4.2)$ by Taylor’s series method.

2. Using Euler’s method solve the initial value problem, $\frac{dy}{dx} - 1 = y^2$, $y(0) = 1$. Obtain $y(0.1), y(0.2)$ and $y(0.3)$.

   Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$ with the initial condition $y = 0$ when $x = 0$, use Picard’s method to obtain $y$ for $x = 0.25, 0.5, 1.0$.

3. Solve the equation $\frac{dy}{dx} = -y$ with $y(0) = 1$ by using Euler’s method.