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AC-3101

B. Sc. (Sem. IV) Examination

April / May – 2015

Mathematics - MTH - 403

(Numerical Analysis - II)

Time : Hours] [Total Marks : 70

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. SC. (SEM. IV)**

Name of the Subject :

☛ **MATHEMATICAL - MTH - 403**

☛ Subject Code No. : **3 1 0 1** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Answer all questions.
(3) Figures to the right indicate full marks of the question.
(4) Follow usual notations.
(5) Use of Scientific non-programmable calculator is allowed.

Que. 1 (a) Answer any **FIVE** as directed : [10]

(1) Use Lagrange's Interpolation formula to obtain the function $f(x)$ for the given data $(0,-1)$ and $(3,2)$.

(2) If $(x) = \frac{1}{x^2}$, then find the divided difference $[x_0\ ,\ x_1\ , x_2]$.

(3) If $b - a = c - b = h,\ h > 0$, then find $[a\ , b, c]$.

(4) Prove : $[x_0,\ x_1] = [x_1,\ x_0]$.

(5) Construct divided difference table:

x:	-2	-1	2	3
$y(x)$:	-12	-8	3	5

(6) What is the necessary condition for applying the Simpson's Rule? Why?

(7) Write the formula to obtain the first derivative at $x = x_n$ and second derivative at $x = x_0$.

(8) Define the Initial Value Problem.

Que.2 (a) Derive Newton's Divided Difference Interpolation formula. [05]

OR

(a) Derive Lagrange's Interpolation formula. [05]

Que.2 (b) Attempt any **TWO** : [10]

(1) Applying Lagrange's Interpolation Formula, find a cubic polynomial which approximates the following data:

x:	-2	-1	2	3
$y(x)$:	-12	-8	3	5

(2) Express the rational function $f(x) = \frac{3x^2+x+1}{(x-1)(x-2)(x-3)}$ as a sum of partial fraction.

(3) Consider the following data, use Newton's Divided Difference Interpolation formula to obtain $\log_{10} 301$:

x:	300	304	305	307
$\log_{10} x$:	2.4771	2.4829	2.4843	2.4871

(4) Using the following table, find $f(x)$ as a polynomial in x using Newton's Divided Difference interpolation formula :

x:	-1	0	3	6	7
$f(x)$:	3	-6	39	822	1611

Que.3 (a) In usual notations prove that, [05]

$$\left[\frac{d^2y}{dx^2}\right]_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right]$$

OR

(a) In usual notation prove that, [05]

$$\left[\frac{dy}{dx}\right]_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

Que.3 (b) Attempt any **TWO** : [10]

(1) The distances travelled by a train at different times are as given below:

t:	0	1	2	3	4	5
s:	0	3	7	15	38	50

Estimate train's velocity for $t = 0$.

(2) The function $y = f(x)$ is defined as follows:

x:	1.00	1.05	1.10	1.15	1.20	1.25	1.30
$f(x)$:	1.00	1.025	1.049	1.072	1.095	1.118	1.140

Compute the values of second derivative at $x = 1.05$

(3) From the following table of values of x and y, obtain the value of the first derivative when $x = 2.2$:

x:	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y:	2.7183	3.3201	4.0532	4.9530	6.0496	7.3891	9.0250

(4) The following table of values x and y is given, find $\frac{d^2y}{dx^2}$ when $x = 6$:

x:	0	1	2	3	4	5	6
y:	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

Que.4 (a) State and prove Simpson's $\frac{1}{3}$ Rule. [05]

OR

(a) State and prove Simpson's $\frac{3}{8}$ Rule. [05]

Que.4 (b) Attempt any **TWO** : [10]

(1) Find the value of $\int_3^7 x^2 \log x \, dx$ by using Trapezoidal Rule by taking 4 subintervals of $[3, 7]$.

(2) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta$ using Simpson's $\frac{1}{3}$ rule with $h = \frac{\pi}{12}$.

(3) Estimate the value of the integral $\int_1^3 \frac{1}{x} dx$ by Simpson's rule with 8 strips.

(4) Evaluate the integral $\int_0^1 \sqrt{1 - x^2} dx$ by taking $h = \frac{1}{6}$.

Que.5 (a) Explain Euler's method to solve the initial value problem $\frac{dy}{dx} = f(x, y)$, where $y(x_0) = y_0$. [05]

OR

(a) Explain Picard's method to solve the initial value problem $\frac{dy}{dx} = f(x, y)$, where $y(x_0) = y_0$. [05]

Que.5 (b) Attempt any **TWO** : [10]

(1) If $\frac{dy}{dx} = \frac{1}{x^2+y}$, $y(4) = 4$, compute the value of $y(4.1), y(4.2)$ by Taylor's series method.

(2) Using Euler's method solve the initial value problem. $\frac{dy}{dx} - 1 = y^2$, $y(0) = 1$. Obtain $y(0.1)$, $y(0.2)$ and $y(0.3)$

(3) Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial condition $y = 0$ when $x = 0$, use Picard's method to obtain y for $x = 0.25, 0.5, 1.0$.

(4) Solve the equation $\frac{dy}{dx} = -y$ with $y(0) = 1$ by using Euler's method.