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AD-3250

Third Year B. Sc. (Sem. VI) Examination

March / April – 2015

Mathematics : Paper - MTH - 602

(Linear Algebra - II)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **T. Y. B. SC. (SEM. 6)**

Name of the Subject :

☛ **MATHEMATICS : MTH - 602**

☛ Subject Code No. : **3 2 5 0** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All quesitons are compulsory.

(3) Figures to the right indicate marks of the questions.

(4) Follow usual notations.

1 Answer the following as directed of any **five** : **10**

(1) Check the linearity of the map $T:V_2 \to V_2$ defined by $T(x, y) = (x^2 + xy, xy)$

(2) Let $T:U \to V$ be a linear map. Then prove that $T(-u) = -T(u)$; for every $u \in U$.

(3) Define : Rank and Nullity of a linear map.

(4) Find $N(T)$ for the linear map $T:V_2 \to V_2$ defined by $T(e_1) = (0,0)$ and $T(e_2) = (0,-1)$.

(5) Prove that the linear map $T:V_2 \to V_2$ defined by $T(e_1) = e_1 - e_2$, and $T(e_2) = e_1 + e_2$ is onto :

(6) Let $T:U \to V$ and $S:V \to W$ be two linear maps. If ST is one-one, then prove that T is one-one.

(7) In an inner product space V; prove that $u \cdot (\propto v) = \overline{\propto}(u.v)$; for all $u, v \in V$ and any scalar $\propto$.

(8) Obtain the vector projection of (0, 0, 1) along the vector (1, 1, 0) in the inner product space V_3.

2 (a) Define a linear map. Check the linearity of the map $T: V_2 \to V_1$ defined by $T(x, y) = x^2 + y^2$. Moreover, If $T: U \to V$ is a linear map, then prove that $T(\theta_U) = \theta_V$. **5**

OR

(a) Let $T: U \to V$ be a linear map. Then prove that $N(T)$ is a subspace of U. **5**

(b) Attempt any **two** : **10**

(1) Prove that a map $T: V_3 \to V_2$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3)$ is a linear.

(2) Let $T: U \to V$ be a linear map. Then prove that $R(T)$ is a subspace of V.

(3) Let $T: U \to V$ be a linear map. Then prove that T is one-one if and only if $N(T) = \{\theta_U\}$.

(4) Obtain the general rule for a linear map $T: V_2 \to V_4$ such that $T(1,1) = (1,1,1,1)$ and $T(1,-1) = (-1,-1,-1,-1)$.

3 (a) Define a non-singular linear map. Let $T: U \to V$ be a linear map. If T is one-one and $u_1, u_2, ..., u_n$ are LI vectors of U, then prove that $T(u_1), T(u_2), ..., T(u_n)$ are also LI. **5**

OR

(a) Let $T: U \to V$ be a linear map and U be a finite-dimensional vector space. Then prove that $\dim R(T) + \dim N(T) = \dim U$. **5**

(b) Attempt any **two** : **10**

(1) Define the Null space of a linear map. Let U and V be finite-dimensional vector spaces of the same dimension. Then prove that a linear map $T:U \to V$ is one-one if and only if T is onto.

(2) Verify the Rank-Nullity theorem for the linear map $T:V_3 \to V_2$ defined by :

$T(e_1)=(2,1)$, $T(e_2)=(0,1)$ and $T(e_3)=(1,1)$

(3) Prove that the linear map $T:V_3 \to V_3$ defined by $T(e_1)=e_1+e_2$, $T(e_2)=e_2+e_3$ and $T(e_3)=e_1+e_2+e_3$ is non-singular and find T^{-1}.

(4) Prove that every real vector space of dimension p is isomorphic to V_p.

4 Attempt any **three** : **15**

(1) Let $T:U \to V$ and $S:V \to W$ be two linear maps. If S and T are non-singular, then prove that ST is non-singular and $(ST)^{-1}=T^{-1}S^{-1}$.

(2) Determine the matrix $(T:B_1,B_2)$ for the linear map $T:V_2 \to V_2$ defined by $T(x,y)=(x,-y)$ relative to the bases :

$B_1=\{(1,1),(1,0)\}$,

$B_2=\{(2,3),(4,5)\}$

(3) Determine the linear map associated with the matrix

$A=\begin{bmatrix}1 & -1 & 2\\ 3 & 1 & 0\end{bmatrix}$ relative to the bases :

$B_1=\{(1,-1,1),(1,2,0),(0,-1,0)\}$, $B_2=\{(1,0),(2,-1)\}$

(4) Verify the Rank-Nullity Theorem for the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$$

(5) Find range, rank, kernel and nullity for the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

5 (a) Define an inner product. In an inner product space V; prove that : **5**

(i) $(u+v)\cdot w = u\cdot w + v\cdot w$,

(ii) $\theta_V \cdot u = 0$.

For all $u, v, w, \in V$.

OR

(a) Prove that an orthogonal set of non-zero vectors in an inner product space is *LI*. **5**

(b) Attempt any **two** : **10**

(1) Define the norm of a vector in an inner product space. In an inner product space V; prove that :

(i) $\|\propto u\| = |\propto| \, \|u\|$

(ii) $\|u\| \geq 0$ and $\|u\| = 0 \Leftrightarrow u = \theta_V$; for every $u \in V$ and any scalar $\propto$.

(2) Let V be an inner product space. Prove that $|u \cdot v| \leq \|u\| \, \|v\|$ for all $u, v \in V$.

(3) State and prove the triangle In equality for the norm on an inner product space.

(4) Orthogonalise the *LI* set $B = \{(1,0,0),(1,1,1),(1,2,3)\}$ of V_3 by the Gram-Schmidt process.