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AD-3251

B. Sc. (Sem. VI) Examination

March/April – 2015

Mathematics : Paper - 603

(Real Analysis - III)

(New Course)

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. SC. (SEM. 6)**

Name of the Subject :

☛ **MATHEMATICS : PAPER - 603 (NEW)**

☛ Subject Code No. : **3 2 5 1** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Digits to the right of each question indicate its marks.

(4) Follows usual symbols.

1 Answer any FIVE from the following. [10]

(1) Prove that if $a_1 + a_2 + ...$ converges to s, then $a_2 + a_3 + ...$ converges to $s - a_1$.

(2) Define convergent series and absolute convergent series.

(3) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

(4) Give statement of RATIO test.

(5) Prove that every finite set is of measure zero.

(6) Define lower sum for a bounded function f on $[a,b]$.

(7) If $f(x) = \int_0^x \sqrt{t + t^5}\, dt$ $(x > 0)$, then find $f'(3)$.

(8) In usual notations prove that $\overline{\int_a^b} f \geq \underline{\int_a^b} f$.

2 (a) State and prove the Leibnitz test for the convergence of an alternating Series. [5]

OR

(a) If $\sum_{n=1}^{\infty} a_n$ is a convergent series then prove that $\lim_{n\to\infty} a_n = 0$. Is converse true ? Justify your answer. [5]

(b) Answer any TWO from the following. [10]

(1) Classify the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+.....$ as to divergent, conditionally convergent, or absolutely convergent.

(2) Does the series $\sum_{n=1}^{\infty}\left(\frac{n+1}{n+2}\right)$ converge or diverge ? Does the series $\sum_{n=1}^{\infty}\frac{n+1}{10^{10}(n+2)}$ converge or diverge ?

(3) Does the series $\sum_{n=1}^{\infty}\log\left(1+\frac{1}{n}\right)$ converge or diverge ?

(4) Prove that the series $(1-2)-\left(1-2^{1/2}\right)+\left(1-2^{1/3}\right)-\left(1-2^{1/4}\right)+....$ converges.

3 (a) Prove that (1) If $\sum_{n=1}^{\infty}|b_n|<\infty$ and if $\lim_{n\to\infty}\frac{|a_n|}{|b_n|}$ exists, then $\sum_{n=1}^{\infty}|a_n|<\infty$. [5]

(2) If $\sum_{n=1}^{\infty}|a_n|=\infty$ and if $\lim_{n\to\infty}\frac{|a_n|}{|b_n|}$ exists, then $\sum_{n=1}^{\infty}|b_n|=\infty$.

OR

(a) If $\{a_n\}_{n=1}^{\infty}$ is a nonincreasing sequence of positive numbers and if $\sum_{n=1}^{\infty}a_n$ converges then prove that $\lim_{n\to\infty} na_n=0$. [5]

(b) Answer any TWO from the following. [10]

(1) Using appropriate TEST of convergence check the convergence for the series $\sum_{n=1}^{\infty}\frac{n^n}{n!}$.

(2) Check the convergence for the series $\sum_{n=1}^{\infty}\frac{3}{4+2^n}$.

(3) For what values of x does $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^x}$ converge ?

(4) For what values of x does $\sum_{n=1}^{\infty}\frac{x^n}{n}$ converge ?

4 (a) If each of the subsets $E_1, E_2, E_3,...$ of R^2 is of measure zero, then prove that $\bigcup_{n=1}^{\infty}E_n$ is also of measure zero. [5]

OR

(a) If $f \in R[a,b]$ and $\lambda \le 0$ is real number, then prove that [5]

$\lambda f \in R[a,b]$ and $\int_a^b \lambda f = \lambda \int_a^b f$.

(b) Answer any TWO from the following. [10]

(1) Let $f(x)=x \ (0 \le x \le 1)$ and $\sigma = \{0, 1/3, 2/3, 1\}$ be any subdivision of $[0,1]$ then compute $L[f;\sigma]$ and $U[f;\sigma]$.

(2) Evaluate :- (i) $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \left(\frac{3}{n}\right)^2 + \dots + \left(\frac{n}{n}\right)^2\right]$;

(ii) $\lim_{n\to\infty} \frac{1}{n}\left(e^{3/n} + e^{6/n} + e^{9/n} + \dots\dots + e^3\right)$.

(3) Prove that continuous function on the closed bounded interval $[a,b]$ is Riemann Integrable.

(4) If A is not of measure zero, if $B \subset A$, and if B is of measure zero then prove that $A-B$ is not of measure zero. Use it to prove that the set of all irrational numbers is not of measure zero.

.5 (a) If f is a continuous function on the closed bounded interval $[a,b]$, and if $\Phi'(x) = f(x) \ (a \le x \le b)$, then prove that [5]

$\int_a^b f(x)\,dx = \Phi(b) - \Phi(a)$.

OR

(a) If f is a continuous $[a,b]$, and if $F(x) = \int_a^x f(t)\,dt \quad (a \le x \le b)$, then prove that F is also continuous $[a,b]$.

(b) Answer any TWO from the following. [10]

(1) If $f \in R[a,b]$, then prove that $|f| \in R[a,b]$ and $\left|\int_a^b f\right| = \int_a^b |f|$.

(2) If f' and g' are continuous on $[a,b]$, then prove that

$\int_a^b f(x)\,g'(x)\,dx = f(b)g(b) - f(a)g(a) - \int_a^b f'(x)\,g(x)\,dx$.

(3) If f is a continuous $[a,b]$, then prove that there exists $c \in (a,b)$ such that $\int_a^b f(x)dx = f(c)(b-a)$.

(4) In usual notations prove that $\frac{1}{3\sqrt{2}} \le \int_0^1 \frac{x^2}{\sqrt{1+x}}\,dx \le \frac{1}{3}$.
