AD-3252

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B. Sc. (Sem. VI) (Mathematics) Examination

March/April – 2015

MTH-604 : Real Analysis - IV

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. SC. (SEM. 6) (MATHEMATICS)**

Name of the Subject :
☛ **MTH-604 : REAL ANALYSIS - 4**

☛ Subject Code No. : **3 2 5 2** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Digits to the right of each question indicate its marks.
(4) Follow usual symbols.
(5) Use of scientific non-programmable is permissible.

1 Answer any five from the following : **10**

(1) Define closure of a set E and in usual notations show that $E \subset \overline{E}$.

(2) Give an example of a set E such that both E and its complement are dense in R^1. Can E be closed ?

(3) Show that the interval $[0,1]$ is not a connected subset of R_d.

(4) Define :
 (i) Bounded set A in a metric space M;
 (ii) Diameter of a set A.

(5) Give statement of "Nested interval theorem".

(6) Give definition of a contraction on a metric space M.

(7) Show that a closed subset of a compact metric space is compact.

(8) Define :
 (i) Open covering of a metric space M;
 (ii) Heine Borel Property.

2 (a) Let E be a subset of the metric space M. Then show that the point $x \in M$ is a limit point of E if every open ball $B[x;r]$ about x contains at least one point of E. **5**

OR

(a) Let $\langle M_1, \rho_1 \rangle$ and $\langle M_2, \rho_2 \rangle$ be metric spaces and let $f: M_1 \to M_2$. Then show that f is continuous of M_1 if and only if $f^{-1}(F)$ is a closed subset of M_1 whenever F is a closed subset of M_2. **5**

(b) Answer any two from the following : **10**

(1) (i) Show that any finite subset of a metric space M is closed.

(ii) If $a \in R^1$, then show that $[a, \infty]$ is a closed subset of R^1.

(2) If F_1 and F_2 are closed subsets of a metric space M, then show that $F_1 \cup F_2$ is also closed.

(3) Let f be a continuous real-valued function on the metric space M. Let A be the set of all $x \in M$ such that $f(x) \geq 0$. Then show that A is closed.

(4) If E is closed subsets of a metric space M, then show that $\overline{E}$ is closed.

3 (a) Show that the range of a continuous function defined on a connected metric space is also connected. **5**

OR

(a) If a subset A of the metric space $\langle M, \rho \rangle$ is Totally bounded then show that A is bounded. **5**

(b) Answer any two from the following : **10**

(1) If A is a connected subset of the metric space M, and if $A \subset B \subset \overline{A}$, then show that B is connected.

(2) Show that every finite subset of a metric space M is Totally bounded.

(3) Show that a subset A of R_d is totally bounded if and only if A contains only a finite number of points.

(4) If A_1 and A_2 are connected subsets of a metric space M, and if $A_1 \cap A_2 \neq \phi$, then show that $A_1 \cup A_2$ is also connected.

4 (a) If $\langle M, \rho \rangle$ is a complete metric space and A is a closed subset of M then show that $\langle A, \rho \rangle$ is also complete. **5**

OR

(a) Show that R^2 is complete. **5**

(b) Answer any two from the following : **10**

(1) Show that the interval $(0, 1)$ with absolute value metric is not a complete metric space, but it is complete with the metric of R_d.

(2) If $T(x) = x^2; \left(0 \leq x \leq \frac{1}{3}\right)$, then show that T is a contraction on $\left[0, \frac{1}{3}\right]$.

(3) (i) Give statement of "Picard's fixed point theorem".

(ii) Show that R_d is complete.

(4) If $T : [0, 1] \to [0, 1]$ and if there is a real number α with $0 \leq \alpha \leq 1$ such that $|T'(x)| \leq \alpha; (0 \leq x \leq 1)$ where T' is the derivative of T, then show that T is a contraction on $[0, 1]$.

5 (a) If the metric space M has the Heine-Borel property then show that M is compact. **5**

OR

(a) Let A be subset of the metric space $\langle M, \rho \rangle$, and if $\langle A, \rho \rangle$ is compact then show that A is closed subset of $\langle M, \rho \rangle$. **5**

(b) Answer any two from the following : **10**

(1) Show that the metric space $\langle M, \rho \rangle$ is compact if and only if every sequence of points in M has a subsequence converging to a point in M.

(2) Show that the metric space $\langle M, \rho \rangle$ is compact if and only if, whenever $\mathscr{F}$ is a family of closed subsets of M with Finite Intersection property, then $\bigcap_{F \in \mathscr{F}} F \neq \phi$.

(3) Show that a connected subset of R_d is compact.

(4) If A and B are compact subsets of R^1, then show that $A \times B$ is a compact subset of R^2.