AD-3253

Third Year B. Sc. (Sem. VI) Examination

March/April – 2015

Mathematics

(MTH-605 : Discrete Mathematics)

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.	
Fillup strictly the details of ☛ signs on your answer book.	Seat No. :
Name of the Examination : ☛ THIRD YEAR B. Sc. (SEM. 6)	
Name of the Subject : ☛ MATHEMATICS	
☛ Subject Code No. : 3 2 5 3 ☛ Section No. (1, 2,.....) : Nil	Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks.

(4) Follow usual notations.

1 Answer the following : (any **five**) **10**

(1) If $x=\{1,2,3,4\}$ and $R=\{\langle 1,1\rangle,\langle 1,4\rangle,\langle 4,1\rangle,\langle 4,4\rangle,\langle 2,2\rangle,\langle 2,3\rangle,\langle 3,2\rangle,\langle 3,3\rangle\}$, then write the matrix R and sketch its graph.

(2) Let $L=\langle 0,1\rangle$ then draw the hasse diagram of $\langle L,\leq\rangle$ and $\left\langle L^3,\leq_3\right\rangle$.

(3) In a Boolean algebra $\langle B,*,\oplus,',0,1\rangle$. Prove that $(a*b)\oplus(a*b')=a:a,b\in B$.

(4) If $\langle B,*,\oplus,1,0,1\rangle$ is a Boolean algebra then show that : $S\subseteq B$ is closed with respect to the set of operations $\{\oplus,1\}$ then it is also closed with respect to the operation.

(5) In any Boolean algebra, show that $a \le b \Rightarrow a + bc = b(a+c)$.

(6) Obtain the product of sums canonical form of $x_1 * x_2$.

(7) Let $\langle L, *, \oplus \rangle$ be a distributive lattice for any $a, b, c \in L$ show that $(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c$.

(8) Show that the operation of join on a lattice is commutative and idempotent.

2 (a) Let I_+ be the set of all positive integers and Let D denotes the relation of 'division' in I_+, such that for any $a, b \in I_+$, aD_b if and only if 'a divides b'. Then show that $\langle I_+, D \rangle$ is a lattice with $a \oplus b = LCM\{a, b\}$ and $a * b = GCD\{a, b\}$. **5**

OR

(a) If the relation R and S are reflexive and symmetric then show that $R \cap S$ is also reflexive and symmetric. **5**

(b) Answer the following : (any **two**) **10**

(1) Define lattice. Let $\langle L, \le \rangle$ is a lattice in which the meet and join are $*$ and $\oplus$ respectively. For any $a, b \in L$, show that $a \le b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

(2) State and prove distributive inequalities in a lattice.

(3) Define : Poset, Irreflexive relation, symmetric relation, well ordered set, equivalence relation.

(4) Let n be a +ve integer and S_n be the set of all divisors of n. Let D denotes the relation of 'division' then draw the Hasse diagram of $\langle S_{30}, D \rangle$ and $\langle S_{60}, D \rangle$.

3 (a) Define : Sublattice, Distributive lattice, lattice homomorphism, Bounded lattice, complete lattice. 5

OR

(a) Prove that the direct product of two distributive lattices is distributive. 5

(b) Answer the following : (any **two**) 10

(1) State and Prove De-Morgan's laws in a complemented distributive lattice.

(2) Define : Lattice isomorphism. Let $\langle L, *, \oplus \rangle$ and $\langle S, \wedge, \vee \rangle$ be two lattices. with partial ordering relation $\leq$ and $\leq'$ respectively. If $g: L \rightarrow S$ is an isomorphism then show that $a \leq b \Leftrightarrow g(a) \leq' g(b)$.

(3) Find the complements of every element of the lattice $\langle S_n, D \rangle$ for $n = 75$ and $n = 30$.

(4) Show that : In a distributive lattice, $(a*b) \oplus (b*c) \oplus (c*a) = (a \oplus b)*(b \oplus c)*(c \oplus a)$

4 (a) Let S be a non-empty set and $e(s)$ be its power set. Then show that $\langle e(s), \cap, \cup, ', \phi, s \rangle$ is a Boolean algebra. 5

OR

(a) Define : Boolean isomorphism, Boolean algebra, Direct product of two Boolean algebras, equivalent Boolean forms. 5

(b) Answer the following : (any **two**) 10

(1) Obtain the sum of products canonical form of $(x_1 \oplus x_2)' \oplus \left(x_1^1 * x_3\right)$.

(2) Define : Sub-Boolean algebra. If $\langle B, *, \oplus, ', 0, 1 \rangle$ is a boolean algebra then show that $S \subseteq B$ is sub-boolean algebra if S is closed with respect to the set of operations $\{\oplus, '\}$

(3) Simplify :

(i) $(a*b)'\oplus(a\oplus b)'$

(ii) $(a*c)\oplus c\oplus[(b\oplus b')*e]$

(4) Check whether the following Boolean expressions are equivalent to one another or not :

(i) $(x\oplus y)*(x'\oplus z)*(y\oplus z)$

(ii) $(x*z)\oplus(x'*y)$

5 (a) Give Cubic and truthtable representation of the Boolean function $f=\overline{w}+y(\overline{x}+\overline{z})$. **5**

OR

(a) Give circuit diagram and Karnaush map representation of the Boolean function : **5**

$f=\overline{x}\,\overline{y}\,z+\overline{x}\,y\,\overline{z}+x\,y\,\overline{z}$.

(b) Answer the following : (any **two**) **10**

(1) Use the Karnaush map representation to find a minimal sum of products expression of the function $f(a,b,c,d)=\sum(0,5,7,8,12,14)$.

(2) Minimize : $f(a,b,c)=\sum(0,1,4,6)$ by using Karnaush map.

(3) Use the Quine-Mc-Cluskey method to find a minimal sum of products expression of the function :

$f(a,b,c,d)=\sum(0,2,3,7)$

(4) Minimize : $\sum(0,2,6,7,8,9,13,15)$ by using Quine-Mc-Cluskey method.