AD-3254

B. Sc. (Sem. VI) Examination

March/April – 2015

Mathematics : MTH-606

(Number Theory - II)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. Sc. (SEM. 6)**

Name of the Subject :
☛ **MATHEMATICS : MTH-606**

☛Subject Code No. : **3 2 5 4** ☛Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Use of non-programmable scientific calculator is allowed.

(3) All the questions are compulsory.

(4) Figures to the right indicate full marks of the question.

(5) Follow usual notations.

1 Answer any five questions : **10**

(1) State the necessary and sufficient condition for the solution of the linear congruence $ax \equiv b(\mod n)$.

(2) Find the value of $\sum_{d|100} 1$

(3) Give an example of a pseudoprime.

(4) Obtain the highest power of 7 which divides 100!

(5) For $k \geq 2$, prove that $n = 2^{k-1}$ satisfies the equation $\sigma(n) = 2n$.

(6) Find $\phi(780)$.

2 (a) State and prove Chinese Remainder Theorem. **5**

OR

(a) Find the smallest integer $a > 2$ such that **5**

$2|a, 3|a+1, 4|a+2, 5|a+3, 6|a+4$

(b) Attempt any **two** : **10**

(1) Solve the linear congruence $18x \equiv 30 \pmod{42}$.

(2) Solve the linear congruence $36x \equiv 8 \pmod{102}$.

(3) Solve the following set of simultaneous congruences :

$x \equiv 1 \pmod 3, x \equiv 2 \pmod 4, x \equiv 3 \pmod 5$

(4) Solve the simultaneous congruences :

$x \equiv 1 \pmod 3, x \equiv 2 \pmod 5, x \equiv 3 \pmod 7$

3 (a) Let p be a prime and suppose that $p \dagger a$. Show that **5**

$a^{p-1} \equiv 1 \pmod p$.

OR

(a) If p and q are distinct primes, then prove that **5**

$a^{pq} - a^p - a^q + a \equiv 0 \pmod{pq}$

(b) Solve any **two** : **10**

(1) Find the remainder when 7^{1947} is divided by 17.

(2) Without using Wilson's theorem verify that

$(p-1)! \equiv -1 \pmod p$, where $P = 7$.

(3) Verify that $4(29!) + 5!$ is divisible by 31.

(4) If p is a prime, then prove that

$$1^{p-1} + 2^{p-1} + 3^{p-1} + \ldots.. + (p-1)^{p-1} \equiv -1 \pmod p$$

4 (a) If $n = p_1^{k_1} p_2^{k_2} \ldots\ldots p_r^{k_r}$ is the prime factorization of $n > 1$, **5**

then prove that $\sigma(n) = \dfrac{p_1^{k_1+1}-1}{p_1-1} \dfrac{p_2^{k_2+1}-1}{p_2-1} \cdots\cdots\cdots \dfrac{p_r^{k_r+1}-1}{p_r-1}$

and $\tau(n) = (k_1+1)(k_2+1)\ldots\ldots\ldots(k_r+1)$.

OR

(a) For any integer $n \geq 3$ show that $\sum_{k=1}^{n} \mu(k!) = 1$. **5**

(b) Attempt any **two** : **10**

(1) For $n = 280$, find $\tau(n)$ and $\sigma(n)$.

(2) Prove that $\tau(n)$ is an odd integer if and only if n is a perfect square.

(3) If 2^{k-1} is prime, then show that $n = 2^{k-1}\left(2^k - 1\right)$ satisfies the equation $\sigma(n) = 2n$.

(4) Find the number of zeros with which 1000! terminates.

5 (a) State and prove Euler's theorem. **5**

OR

(a) For n > 2, prove that $\phi(n)$ is an even integer. **5**

(b) Solve any two : **10**

(1) Use Euler's theorem to find the last two digits in the decimal expansion of 3^{243}.

(2) If the integer $n > 1$ has r distinct odd prime factors, then prove that $2^r \mid \phi(n)$.

(3) Prove that

$$\phi(2n) = \phi(n) \text{ if } n \text{ is odd.}$$

$$= 2\phi(n) \text{ if } n \text{ is even.}$$

(4) If p is prime and $k \geq 2$ then show that

$$\phi\left(\phi\left(p^k\right)\right) = p^{k-2}\phi\left((p-1)^2\right).$$