1. Answer the following: (any five)  

(1) State Parseval's identities for Fourier cosine transform.

(2) State one dimensional heat-flow equation.

(3) Write the complex form of Fourier Integral and also write inverse Fourier Transform of \( f(x) \).

(4) Define Convolution of two functions.

(5) Prove that \( F \left[ \frac{\partial^2 u}{\partial x^2} \right] = -s^2 F[u] \).

(6) State the change of scale property for Fourier sine transform.

(7) Define Fourier transform.

(8) State one dimensional wave equation for vibrating string.
2  (a) State and prove shifting property of Fourier Transform.  
   
   OR  
   
   (a) Derive Fourier sine and cosine integral.  
   
   (b) Attempt any two:  
   
   (1) Solve the integral equation  
   \[ \int_0^\infty f(\theta) \cos \alpha \theta d\theta = \begin{cases} 
   1 - \alpha; & 0 \leq \alpha \leq 1 \\
   0; & \alpha > 1 
   \end{cases} \]. Hence evaluate  
   \[ \int_0^\infty \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2} \]  
   
   (2) Find the Fourier transform of  
   \[ f(x) = \begin{cases} 
   1; & |x| < 1 \\
   0; & |x| > 1 
   \end{cases} \]. Hence evaluate  
   \[ \int_0^\infty \frac{\sin x}{x} dx \].  
   
3  (a) State and prove convolution theorem for Fourier Transform.  
   
   OR  
   
   (a) State Parseval’s identity for Fourier Transform and prove it.  
   
   (b) Attempt any two:  
   
   (1) Using Parseval’s identity, prove that  
   \[ \int_0^\infty \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)} \]  
   
   (2) Using Parseval’s identity, If  
   \[ f(x) = \begin{cases} 
   1; & 0 < x < 1 \\
   0; & x > 1 
   \end{cases} \]  
   then evaluate  
   \[ \int_0^\infty \frac{\sin^2 x}{x^2} dx \].
(a) Derive Fourier transform of the derivatives of a function.

OR

(a) Determine the distribution of temperature in the semi infinite medium \( x \geq 0 \), when the end \( x = 0 \) is maintained at zero temperature and the initial distribution of temperature is \( f(x) \).

(b) Attempt any two:

(1) Using the Method of residues, evaluate

\[
L^{-1}\left\{\frac{1}{s^2\left(s^2-a^2\right)}\right\}.
\]

(2) Using the method of residues, evaluate

\[
L^{-1}\left\{\frac{1}{(s+1)(s-1)^2}\right\}.
\]