

**A**

DF-3014
B. Sc. (Sem. III) Examination
March / April - 2016
Mathematics : Paper - MTH-301
(Advanced Calculus - I)
(New Course)

Time : 2 Hours]

[Total Marks : 50

Instructions :

(1)

<p>નીચે દર્શાવેલ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fillup strictly the details of signs on your answer book.</p> <p>Name of the Examination :</p> <p>← B. Sc. (SEM. 3)</p> <p>Name of the Subject :</p> <p>← MATHEMATICS - MTH-301 (NEW COURSE)</p> <p>← Subject Code No. : 3 0 1 4 ← Section No. (1, 2,.....): Nil</p>	<p>Seat No. :</p> <table border="1" style="width: 100%; height: 20px;"><tr><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td><td style="width: 15%;"></td></tr></table> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; text-align: center; margin-top: 10px;">Student's Signature</div>						

(2) There are four sections A, B, C, D in this question paper having 18 questions.

Section A : Question No. 1 to 4 each of 1 mark.

Section B : Question No. 5 to 8 each of 2 marks.

Section C : Question No. 9 to 14 each of 3 marks.

Section D : Question No. 15 to 18 each of 5 marks.

(3) There is only one correct answer for each question.

(4) Follow usual notations.

(5) Use of non-programmable scientific calculator is allowed.

O.M.R. Sheet ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ
O.M.R. Sheet-ની પાછળ છાપેલ છે.
Important instructions to fillup O.M.R. Sheet
is given back side of provided O.M.R. Sheet.

SECTION - A

Question No. 1 to 4 each of 1 mark.

1 $\lim_{y \rightarrow 0} \left\{ \lim_{x \rightarrow 0} \frac{xy}{\sqrt{x^2 + y^2}} \right\} = \underline{\hspace{2cm}}$

- (A) 0
- (B) 1
- (C) -1
- (D) 2

2 Maclaurin's expansion for function of two variables is _____

(A) $f(x, y) = f(0, 0) + \sum_{r=1}^n \frac{1}{r!} \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^r$

$$f(0, 0) + \frac{1}{n!} \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^n f(\theta x, \theta y), \theta \in (0, 1)$$

(B) $f(x, y) = f(0, 0) + \sum_{r=1}^{n-1} \frac{1}{r!} \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^r$

$$f(0, 0) + \frac{1}{n!} \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^n f(\theta x, \theta y), \theta \in (0, 1)$$

(C) $f(x, y) = f(a, b) + \sum_{r=1}^n \frac{1}{r!} \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^r$

$$f(a, b) + \frac{1}{n!} \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^n f(\theta x, \theta y), \theta \in (0, 1)$$

(D) $f(x+h, y+k) = f(x, y) + \sum_{r=1}^{n-1} \frac{1}{r!} \left(h \frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right)^r$

$$f(x, y) + \frac{1}{n!} \left(h \frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right)^n f(x+\theta h, y+\theta k), \theta \in (0, 1)$$

3 $\int_1^2 \int_0^1 x^2 y \, dx dy = \underline{\hspace{2cm}}$

(A) $\frac{1}{6}$

(B) $-\frac{1}{6}$

(C) $\frac{7}{6}$

(D) $\frac{9}{6}$

4 If $\vec{r} = (1 - \cos t)\hat{i} + (t - \sin t)\hat{j} + (t^3 + t^2 + t + 1)\hat{k}$, then $\frac{d\vec{r}}{dt} = \underline{\hspace{2cm}}$

(A) $\sin t\hat{i} + (1 - \cos t)\hat{j} + (3t^2 - 2t + 1)\hat{k}$

(B) $\sin t\hat{i} + (1 - \cos t)\hat{j} + (3t^2 + 2t + 1)\hat{k}$

(C) $-\sin t\hat{i} + (1 - \cos t)\hat{j} + (3t^2 + 2t + 1)\hat{k}$

(D) $\sin t\hat{i} + (1 + \cos t)\hat{j} + (3t^2 + 2t + 1)\hat{k}$

SECTION - B

Question No. 5 to 8 each of 2 marks.

5 If $f(x, y) = \frac{x-y}{x+y}$, $x+y \neq 0$, then f_y at point $(x, y) = (2, -1)$ is _____
 $= 0$, $x+y=0$

- (A) -2
- (B) -4
- (C) 0
- (D) does not exist

6 If $x^2 + y^2 = r^2$ and $\tan \theta = \frac{y}{x}$ then $\frac{\partial(r, \theta)}{\partial(x, y)} = \text{_____}$

- (A) 1
- (B) r
- (C) $1/r$
- (D) r^2

7 $\int_1^2 \int_0^y \frac{dx dy}{x^2 + y^2} = \text{_____}$

- (A) $\frac{\pi}{2} \log 4$
- (B) $\frac{\pi}{4} \log 4$
- (C) $\frac{\pi}{2} \log 2$
- (D) $\frac{\pi}{4} \log 2$

8 If $f = x^2y + y^2x + z^2$, then the value of ∇f at point $(1, 0, -2)$ is _____

- (A) $\hat{i} - 4\hat{k}$
- (B) $\hat{i} + 4\hat{k}$
- (C) $\hat{j} - 4\hat{k}$
- (D) $\hat{j} + 4\hat{k}$

SECTION - C

Question No. 9 to 14 each of 3 marks.

9 If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ _____.

(A) $\frac{1}{4}\tan u$

(B) $\frac{1}{2}\cot u$

(C) $\frac{1}{2}\tan u$

(D) $\frac{1}{2}\sec u$

10 If $u^3 + v^3 = x + y$, $u^2 + v^2 = x^3 + y^3$ then $\frac{\partial(u, v)}{\partial(x, y)} =$ _____

(A) $-\frac{1}{2}\frac{y^2 - x^2}{uv(u-v)}$

(B) $\frac{1}{2}\frac{y^2 + x^2}{uv(u-v)}$

(C) $-\frac{1}{2}\frac{y^2 - x^2}{uv(u+v)}$

(D) $\frac{1}{2}\frac{y^2 - x^2}{uv(u-v)}$

11 If S is a region bounded by the line $y = 3x$, x -axis and the line $x = 6$

then $\iint_S x^2 y^2 dx dy =$ _____

(A) $\frac{3}{2} \times 6^5$

(B) $\frac{3}{2} \times 6^6$

(C) $\frac{2}{3} \times 6^6$

(D) $\frac{5}{2} \times 6^5$

12 If $\phi = xy^2z$ and $\vec{f} = xz\hat{i} - xyj + yz^2k$, then $\frac{\partial^3}{\partial x^2 \partial z}(\phi \vec{f})$ at $(2, -1, 1)$ is

(A) $4y^2z\hat{i} - 2y^3\hat{j}$

(B) $4y^2z\hat{i} + 2y^3\hat{j}$

(C) $4\hat{i} + 2\hat{j}$

(D) $4\hat{i} - 2\hat{j}$

13 $\int_0^\pi \int_0^{a(1+\cos\theta)} r^2 \sin\theta \cos\theta d\theta dr =$ _____

(A) $\frac{a^2}{5}$

(B) $\frac{8a^2}{5}$

(C) $\frac{4a^3}{5}$

(D) $\frac{8a^3}{15}$

14 If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$, then $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} =$ _____

(A) $r^2 \sin\theta$

(B) $r \sin\theta$

(C) $r^2 \cos\theta$

(D) r^2

SECTION - D

Question No. 15 to 18 each of 5 marks.

15 If $u = f(r)$, $r^2 = x^2 + y^2 + z^2$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} =$ _____

(A) $f'(r) - \frac{2}{r} \cdot f''(r)$

(B) $f''(r) - \frac{2}{r} \cdot f'(r)$

(C) $f'(r) + \frac{2}{r} \cdot f''(r)$

(D) $f''(r) + \frac{2}{r} \cdot f'(r)$

16 Expression of $f(x, y) = \frac{y^2}{x^3}$ in powers of $(x-1)$ and $(y+1)$ is _____.

(A) $1 - [3x + 2y - 1] + [6x^2 + y^2 + 6xy - 6x - 4y + 1] + \dots$

(B) $1 - [3(x-1) + 2(y+1)] + [6(x-1)^2 + 6(x-1)(y+1) + (y+1)^2] + \dots$

(C) $1 - [3(x-1) + 2(y+1)] + [6(x-1)^2 + (x-1)(y+1) + (y+1)^2] + \dots$

(D) $1 - [3(x-1) + 2(y+1)] + [6(x-1)^2 + (x-1)(y+1) + 2(y+1)^2] + \dots$

17 $\int_0^{r \cos \theta} \int_{x \tan \theta}^{\sqrt{r^2 - x^2}} f(x, y) dx dy = \underline{\hspace{2cm}}$

(A) $\int_0^{r \sin \theta} \int_0^{y \cot \theta} f(x, y) dy dx + \int_{r \sin \theta}^r \int_0^{\sqrt{r^2 - y^2}} f(x, y) dy dx$

(B) $\int_0^{r \sin \theta} \int_0^{y \tan \theta} f(x, y) dy dx + \int_{r \sin \theta}^r \int_0^{\sqrt{r^2 - y^2}} f(x, y) dy dx$

(C) $\int_0^{r \sin \theta} \int_0^{y \cot \theta} f(x, y) dy dx + \int_{r \sin \theta}^r \int_0^{\sqrt{r^2 - x^2}} f(x, y) dy dx$

(D) $\int_0^{r \sin \theta} \int_0^{y \cot \theta} f(x, y) dy dx + \int_{r \sin \theta}^r \int_0^{\sqrt{r^2 - y^2}} f(x, y) dy dx$

18 If $\vec{r} = \vec{a} \cos \omega t + \vec{b} \sin \omega t$, where \vec{a} and \vec{b} are constant vectors and

ω is a constant scalar, then $\frac{d^2 \vec{r}}{dt^2}$ and $\vec{r} \times \frac{d \vec{r}}{dt} = \underline{\hspace{2cm}}$ respectively.

(A) $-\omega^2 \vec{r}, \theta(a \times b)$

(B) $\omega^2 \vec{r}, \omega \left(\vec{a} \times \vec{b} \right)$

(C) $0, \omega \left(\vec{a} \times \vec{b} \right)$

(D) $-\omega^2 \vec{r}, \omega \left(\vec{a} \times \vec{b} \right)$