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DF-3014

B. Sc. (Sem. III) Examination

March / April – 2016

Mathematics : Paper - MTH-301

(Advanced Calculus - I)

(New Course)

Time : 2 Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. Sc. (SEM. 3)**

Name of the Subject :
☛ **MATHEMATICS - MTH-301 (NEW COURSE)**

☛ Subject Code No. : **3 0 1 4** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) There are four sections A, B, C, D in this question paper having 18 questions.

Section A : Question No. 1 to 4 each of 1 mark.

Section B : Question No. 5 to 8 each of 2 marks.

Section C : Question No. 9 to 14 each of 3 marks.

Section D : Question No. 15 to 18 each of 5 marks.

(3) There is only one correct answer for each question.

(4) Follow usual notations.

(5) Use of non-programmable scientific calculator is allowed.

O.M.R. Sheet ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ O.M.R. Sheetની પાછળ છાપેલ છે.

Important instructions to fillup O.M.R. Sheet is given back side of provided O.M.R. Sheet.

SECTION - A

Question No. 1 to 4 each of 1 mark.

1 $\int_1^2 \int_0^1 x^2 y \, dxdy =$ _______

(A) $\frac{9}{6}$

(B) $\frac{1}{6}$

(C) $-\frac{1}{6}$

(D) $\frac{7}{6}$

2 If $\vec{r} = (1-\cos t)\hat{i} + (t - \sin t) j + \left(t^3 + t^2 + t + 1\right) k$, then $\frac{d\vec{r}}{dt} =$ _______

(A) $\sin t\,\hat{i} + (1+\cos t)\hat{j} + \left(3t^2 + 2t + 1\right)\hat{k}$

(B) $\sin t\,\hat{i} + (1-\cos t)\hat{j} + \left(3t^2 - 2t + 1\right)\hat{k}$

(C) $\sin t\,\hat{i} + (1-\cos t)\hat{j} + \left(3t^2 + 2t + 1\right)\hat{k}$

(D) $-\sin t\,\hat{i} + (1-\cos t)\hat{j} + \left(3t^2 + 2t + 1\right)\hat{k}$

3 $\lim_{y\to 0}\left\{\lim_{x\to 0}\frac{xy}{\sqrt{x^2+y^2}}\right\}=$ ________

(A) 2
(B) 0
(C) 1
(D) –1

4 Maclaurin's expansion for function of two variables is ________

(A) $f(x+h, y+k)=f(x, y)+\sum_{r=1}^{n-1}\frac{1}{r!}\left(h\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)^r$

$f(x, y)+\frac{1}{n!}\left(h\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)^n f(x+\theta h, y+\theta k), \theta\in(0,1)$

(B) $f(x, y)=f(0,0)+\sum_{r=1}^{n}\frac{1}{r!}\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)^r$

$f(0,0)+\frac{1}{n!}\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)^n f(\theta x, \theta y), \theta\in(0,1)$

(C) $f(x, y)=f(0,0)+\sum_{r=1}^{n-1}\frac{1}{r!}\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)^r$

$f(0,0)+\frac{1}{n!}\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)^n f(\theta x, \theta y), \theta\in(0,1)$

(D) $f(x, y)=f(a,b)+\sum_{r=1}^{n}\frac{1}{r!}\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)^r$

$f(a,b)+\frac{1}{n!}\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)^n f(\theta x, \theta y), \theta\in(0,1)$

SECTION - B

Question No. 5 to 8 each of 2 marks.

5 $\int_{1}^{2}\int_{0}^{y} \frac{dxdy}{x^2+y^2} =$ ________

(A) $\frac{\pi}{4}\log 2$

(B) $\frac{\pi}{2}\log 4$

(C) $\frac{\pi}{4}\log 4$

(D) $\frac{\pi}{2}\log 2$

6 If $f = x^2y + y^2x + z^2$, then the value of ∇f at point $(1, 0, -2)$ is _____

(A) $\hat{j} + 4\hat{k}$

(B) $\hat{i} - 4\hat{k}$

(C) $\hat{i} + 4\hat{k}$

(D) $\hat{j} - 4\hat{k}$

7 If $f(x, y) = \frac{x-y}{x+y}, \quad x+y \neq 0$, then f_y at point $(x, y) = (2, -1)$ is _____

$= 0, \quad x + y = 0$

(A) does not exist
(B) –2
(C) –4
(D) 0

8 If $x^2 + y^2 = r^2$ and $\tan\theta = \frac{y}{x}$ then $\frac{\partial(r, \theta)}{\partial(x, y)} =$ ________

(A) r^2
(B) 1
(C) r
(D) $1/r$

SECTION - C

Question No. 9 to 14 each of 3 marks.

9 If $\phi = xy^2 z$ and $\vec{f} = xz\hat{i} - xyj + yz^2 k$, then $\dfrac{\partial^3}{\partial x^2 \partial z}(\phi \vec{f})$ at $(2,-1,1)$ is ________

(A) $4\hat{i} - 2\hat{j}$

(B) $4y^2 z\hat{i} - 2y^3\hat{j}$

(C) $4y^2 z\hat{i} + 2y^3\hat{j}$

(D) $4\hat{i} + 2\hat{j}$

10 $\displaystyle\int_0^{\pi} \int_0^{a(1+\cos\theta)} r^2 \sin\theta \cos\theta \, d\theta \, dr =$ _____

(A) $\dfrac{8a^3}{15}$

(B) $\dfrac{a^2}{5}$

(C) $\dfrac{8a^2}{5}$

(D) $\dfrac{4a^3}{5}$

11 If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then $\dfrac{\partial(x, y, z)}{\partial(r, \theta, \phi)} =$ _____

(A) r^2

(B) $r^2 \sin\theta$

(C) $r\sin\theta$

(D) $r^2 \cos\theta$

12 If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$ ______.

(A) $\frac{1}{2}\sec u$

(B) $\frac{1}{4}\tan u$

(C) $\frac{1}{2}\cot u$

(D) $\frac{1}{2}\tan u$

13 If $u^3+v^3=x+y,\ u^2+v^2=x^3+y^3$ then $\frac{\partial(u, v)}{\partial(x, y)}=$ ______

(A) $\frac{1}{2}\frac{y^2-x^2}{uv(u-v)}$

(B) $-\frac{1}{2}\frac{y^2-x^2}{uv(u-v)}$

(C) $\frac{1}{2}\frac{y^2+x^2}{uv(u-v)}$

(D) $-\frac{1}{2}\frac{y^2-x^2}{uv(u+v)}$

14 If S is a region bounded by the line $y=3x$, x-axis and the line $x=6$ then $\iint_S x^2y^2dx\,dy=$ ______

(A) $\frac{5}{2}\times 6^5$

(B) $\frac{3}{2}\times 6^5$

(C) $\frac{3}{2}\times 6^6$

(D) $\frac{2}{3}\times 6^6$

SECTION - D

Question No. 15 to 18 each of 5 marks.

15 $\int_{0}^{r\cos\theta}\int_{x\tan\theta}^{\sqrt{r^2-x^2}} f(x, y)\,dxdy =$ ________

(A) $\int_{0}^{r\sin\theta}\int_{0}^{y\cot\theta} f(x, y)\,dydx + \int_{r\sin\theta}^{\sqrt{r^2-y^2}}\int_{0}^{r} f(x, y)\,dy\,dx$

(B) $\int_{0}^{r\sin\theta}\int_{0}^{y\cot\theta} f(x, y)\,dydx + \int_{r\sin\theta}^{r}\int_{0}^{\sqrt{r^2-y^2}} f(x, y)\,dy\,dx$

(C) $\int_{0}^{r\sin\theta}\int_{0}^{y\tan\theta} f(x, y)\,dydx + \int_{r\sin\theta}^{r}\int_{0}^{\sqrt{r^2-y^2}} f(x, y)\,dy\,dx$

(D) $\int_{0}^{r\sin\theta}\int_{0}^{y\cot\theta} f(x, y)\,dydx + \int_{r\sin\theta}^{r}\int_{0}^{\sqrt{r^2-x^2}} f(x, y)\,dy\,dx$

16 If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$, where $\vec{a}$ and $\vec{b}$ are constant vectors and ω is a constant scalar, then $\frac{d^2\vec{r}}{dt^2}$ and $\vec{r}\times\frac{d\vec{r}}{dt} =$ ________ respectively.

(A) $-\omega^2\vec{r}, \omega\left(\vec{a}\times\vec{b}\right)$

(B) $-\omega^2\vec{r}, \theta\left(a\times b\right)$

(C) $\omega^2\vec{r}, \omega\left(\vec{a}\times\vec{b}\right)$

(D) $0, \omega\left(\vec{a}\times\vec{b}\right)$

17 If $u = f(r)$, $r^2 = x^2 + y^2 + z^2$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} =$ ______

(A) $f''(r) + \frac{2}{r} \cdot f'(r)$

(B) $f'(r) - \frac{2}{r} \cdot f''(r)$

(C) $f''(r) - \frac{2}{r} \cdot f'(r)$

(D) $f'(r) + \frac{2}{r} \cdot f''(r)$

18 Expression of $f(x, y) = \frac{y^2}{x^3}$ in powers of $(x-1)$ and $(y+1)$ is ______.

(A) $1 - [3(x-1) + 2(y+1)] + \left[6(x-1)^2 + (x-1)(y+1) + 2(y+1)^2\right] + \ldots$

(B) $1 - [3x + 2y - 1] + \left[6x^2 + y^2 + 6xy - 6x - 4y + 1\right] + \ldots$

(C) $1 - [3(x-1) + 2(y+1)] + \left[6(x-1)^2 + 6(x-1)(y+1) + (y+1)^2\right] + \ldots$

(D) $1 - [3(x-1) + 2(y+1)] + \left[6(x-1)^2 + (x-1)(y+1) + (y+1)^2\right] + \ldots$