DF-3014

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B. Sc. (Sem. III) Examination

March / April – 2016

Mathematics : Paper - MTH-301

(Advanced Calculus - I)

(New Course)

Time : **2** Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. Sc. (SEM. 3)**

Name of the Subject :
☛ **MATHEMATICS - MTH-301 (NEW COURSE)**

☛ Subject Code No. : **3 0 1 4** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) There are four sections A, B, C, D in this question paper having 18 questions.

Section A : Question No. 1 to 4 each of 1 mark.

Section B : Question No. 5 to 8 each of 2 marks.

Section C : Question No. 9 to 14 each of 3 marks.

Section D : Question No. 15 to 18 each of 5 marks.

(3) There is only one correct answer for each question.

(4) Follow usual notations.

(5) Use of non-programmable scientific calculator is allowed.

***O.M.R. Sheet* ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ *O.M.R. Sheet*ની પાછળ છાપેલ છે.**

Important instructions to fillup O.M.R. Sheet is given back side of provided O.M.R. Sheet.

SECTION - A

Question No. 1 to 4 each of 1 mark.

1 Maclaurin's expansion for function of two variables is _________

(A) $$f(x, y) = f(a, b) + \sum_{r=1}^{n} \frac{1}{r!}\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^r$$

$$f(a, b) + \frac{1}{n!}\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^n f(\theta x, \theta y), \theta \in (0, 1)$$

(B) $$f(x+h, y+k) = f(x, y) + \sum_{r=1}^{n-1} \frac{1}{r!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^r$$

$$f(x, y) + \frac{1}{n!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^n f(x+\theta h, y+\theta k), \theta \in (0, 1)$$

(C) $$f(x, y) = f(0, 0) + \sum_{r=1}^{n} \frac{1}{r!}\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^r$$

$$f(0, 0) + \frac{1}{n!}\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^n f(\theta x, \theta y), \theta \in (0, 1)$$

(D) $$f(x, y) = f(0, 0) + \sum_{r=1}^{n-1} \frac{1}{r!}\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^r$$

$$f(0, 0) + \frac{1}{n!}\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^n f(\theta x, \theta y), \theta \in (0, 1)$$

2 $\int_{1}^{2}\int_{0}^{1} x^2 y \, dxdy =$ ______

(A) $\frac{7}{6}$

(B) $\frac{9}{6}$

(C) $\frac{1}{6}$

(D) $-\frac{1}{6}$

3 If $\vec{r} = (1-\cos t)\,\hat{i} + (t-\sin t)\,j + (t^3+t^2+t+1)\,k$, then $\frac{d\vec{r}}{dt} =$ ______

(A) $-\sin t\,\hat{i} + (1-\cos t)\,\hat{j} + (3t^2+2t+1)\,\hat{k}$

(B) $\sin t\,\hat{i} + (1+\cos t)\,\hat{j} + (3t^2+2t+1)\,\hat{k}$

(C) $\sin t\,\hat{i} + (1-\cos t)\,\hat{j} + (3t^2-2t+1)\,\hat{k}$

(D) $\sin t\,\hat{i} + (1-\cos t)\,\hat{j} + (3t^2+2t+1)\,\hat{k}$

4 $\lim_{y\to 0}\left\{\lim_{x\to 0}\frac{xy}{\sqrt{x^2+y^2}}\right\} =$ ______

(A) –1

(B) 2

(C) 0

(D) 1

SECTION - B

Question No. 5 to 8 each of 2 marks.

5 If $x^2 + y^2 = r^2$ and $\tan\theta = \frac{y}{x}$ then $\frac{\partial(r,\theta)}{\partial(x,y)} =$ ________

(A) $1/r$
(B) r^2
(C) 1
(D) r

6 $\int_1^2 \int_0^y \frac{dxdy}{x^2 + y^2} =$ ________

(A) $\frac{\pi}{2}\log 2$

(B) $\frac{\pi}{4}\log 2$

(C) $\frac{\pi}{2}\log 4$

(D) $\frac{\pi}{4}\log 4$

7 If $f = x^2y + y^2x + z^2$, then the value of ∇f at point $(1, 0, -2)$ is _____

(A) $\hat{j} - 4\hat{k}$

(B) $\hat{j} + 4\hat{k}$

(C) $\hat{i} - 4\hat{k}$

(D) $\hat{i} + 4\hat{k}$

8 If $f(x,y) = \frac{x-y}{x+y}, \quad x+y \neq 0$, then f_y at point $(x,y) = (2,-1)$ is _____

$\quad\quad = 0, \quad x + y = 0$

(A) 0
(B) does not exist
(C) –2
(D) –4

SECTION - C

Question No. 9 to 14 each of 3 marks.

9 If S is a region bounded by the line $y = 3x$, x-axis and the line $x = 6$ then $\iint_S x^2 y^2 dx\, dy =$ ______

(A) $\frac{2}{3} \times 6^6$

(B) $\frac{5}{2} \times 6^5$

(C) $\frac{3}{2} \times 6^5$

(D) $\frac{3}{2} \times 6^6$

10 If $\phi = xy^2 z$ and $\vec{f} = xz\,\hat{i} - xyj + yz^2 k$, then $\frac{\partial^3}{\partial x^2 \partial z}(\phi \vec{f})$ at $(2, -1, 1)$ is ________

(A) $4\hat{i} + 2\hat{j}$

(B) $4\hat{i} - 2\hat{j}$

(C) $4y^2 z\,\hat{i} - 2y^3\,\hat{j}$

(D) $4y^2 z\,\hat{i} + 2y^3\,\hat{j}$

11 $\int_0^{\pi} \int_0^{a(1+\cos\theta)} r^2 \sin\theta \cos\theta\, d\theta\, dr =$ ______

(A) $\frac{4a^3}{5}$

(B) $\frac{8a^3}{15}$

(C) $\frac{a^2}{5}$

(D) $\frac{8a^2}{5}$

12 If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} =$ ______

(A) $r^2\cos\theta$

(B) r^2

(C) $r^2\sin\theta$

(D) $r\sin\theta$

13 If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ ______.

(A) $\frac{1}{2}\tan u$

(B) $\frac{1}{2}\sec u$

(C) $\frac{1}{4}\tan u$

(D) $\frac{1}{2}\cot u$

14 If $u^3 + v^3 = x + y$, $u^2 + v^2 = x^3 + y^3$ then $\frac{\partial(u, v)}{\partial(x, y)} =$ ______

(A) $-\frac{1}{2}\frac{y^2 - x^2}{uv(u+v)}$

(B) $\frac{1}{2}\frac{y^2 - x^2}{uv(u-v)}$

(C) $-\frac{1}{2}\frac{y^2 - x^2}{uv(u-v)}$

(D) $\frac{1}{2}\frac{y^2 + x^2}{uv(u-v)}$

SECTION - D

Question No. 15 to 18 each of 5 marks.

15 Expression of $f(x, y)=\frac{y^2}{x^3}$ in powers of $(x-1)$ and $(y+1)$ is _____.

(A) $1-[3(x-1)+2(y+1)]+\left[6(x-1)^2+(x-1)(y+1)+(y+1)^2\right]+....$

(B) $1-[3(x-1)+2(y+1)]+\left[6(x-1)^2+(x-1)(y+1)+2(y+1)^2\right]+....$

(C) $1-[3x+2y-1]+\left[6x^2+y^2+6xy-6x-4y+1\right]+.....$

(D) $1-[3(x-1)+2(y+1)]+\left[6(x-1)^2+6(x-1)(y+1)+(y+1)^2\right]+....$

16 $\int_0^{r\cos\theta}\int_{x\tan\theta}^{\sqrt{r^2-x^2}} f(x, y)\,dxdy =$ _______

(A) $\int_0^{r\sin\theta}\int_0^{y\cot\theta} f(x, y)\,dydx+\int_{r\sin\theta}^{r}\int_0^{\sqrt{r^2-x^2}} f(x, y)\,dy\,dx$

(B) $\int_0^{r\sin\theta}\int_0^{y\cot\theta} f(x, y)\,dydx+\int_{r\sin\theta}^{\sqrt{r^2-y^2}}\int_0^{r} f(x, y)\,dy\,dx$

(C) $\int_0^{r\sin\theta}\int_0^{y\cot\theta} f(x, y)\,dydx+\int_{r\sin\theta}^{r}\int_0^{\sqrt{r^2-y^2}} f(x, y)\,dy\,dx$

(D) $\int_0^{r\sin\theta}\int_0^{y\tan\theta} f(x, y)\,dydx+\int_{r\sin\theta}^{r}\int_0^{\sqrt{r^2-y^2}} f(x, y)\,dy\,dx$

17 If $\vec{r} = \vec{a} \cos \omega t + \vec{b} \sin \omega t$, where $\vec{a}$ and $\vec{b}$ are constant vectors and ω is a constant scalar, then $\frac{d^2 \vec{r}}{dt^2}$ and $\vec{r} \times \frac{d \vec{r}}{dt} =$ ______ respectively.

(A) $0, \omega\left(\vec{a} \times \vec{b}\right)$

(B) $-\omega^2 \vec{r}, \omega\left(\vec{a} \times \vec{b}\right)$

(C) $-\omega^2 \vec{r}, \theta(a \times b)$

(D) $\omega^2 \vec{r}, \omega\left(\vec{a} \times \vec{b}\right)$

18 If $u = f(r)$, $r^2 = x^2 + y^2 + z^2$, then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} =$ ______

(A) $f'(r) + \frac{2}{r} \cdot f''(r)$

(B) $f''(r) + \frac{2}{r} \cdot f'(r)$

(C) $f'(r) - \frac{2}{r} \cdot f''(r)$

(D) $f''(r) - \frac{2}{r} \cdot f'(r)$