* D F - 3 0 1 7 *

DF-3017

Second Year B. Sc. (Sem. III) (Mathematics for Computer Science) Examination

April / May - 2016

MCS-301 : Advanced Calculus-II

(New Course)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ S.Y. B.SC. (SEM. III) (MATH. FOR COM. SCI.)

Name of the Subject :

☛ MCS-301 : ADVANCED CALCULUS-II (NEW COURSE)

☛ Subject Code No. : 3 0 1 7 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All the questions are compulsory.

(3) Digits shown on right hand side indicate full marks of the question.

(4) Symbols have their usual meaning.

Q1 Answer the following questions as directed (10)

(i) If f(x,y) = xy -3x +4 then find limit at the point (3,8)

(ii) Test the convergence of the series $1+\frac{3}{5}+\frac{8}{10}+\frac{15}{17}\ldots\ldots\ldots\ldots+\frac{2^n-1}{2^n+1}+\cdots$

(iii) If f(x,y) = $sin^{-1}\left(\frac{x}{y}\right)+tan^{-1}\left(\frac{y}{x}\right)$ then show that $xu_x + yu_y = 0$

(iv) Evaluate $\int_0^2\int_0^{x^2} y\,dy\,dx$

(v) Evaluate $\int_0^\infty x^4e^{-2x}dx$

Q2 (a) If $f(x,y)=\begin{cases}\frac{3xy^2}{x^2+y^4}, & (x,y)\neq 0\\ 0, & (x,y)=0\end{cases}$ show that f(x,y) is not continuous at (0,0).Further find

f_x and f_y at (1,2) (5)

OR

(a) Find the extreme values of the function f (x,y) = $x^2 + y^2 +6x + 12$

(b) Attempt any two (10)

(i) Expand f(x,y) = sin xy in powers of (x-1) and (y+2) by Taylor's theorem

(ii) If u = log($x^3+ y^3 + z^3$ - 3xyz) then show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-9(x+y+z)^{-2}$

(iii) If $f(x,y)=x\emptyset\left(\frac{y}{x}\right)+y\varphi\left(\frac{x}{y}\right)$ then prove that $x^2\frac{\partial^2 f}{\partial x^2}+2xy\frac{\partial^2 f}{\partial x\partial y}+y^2\frac{\partial^2 f}{\partial y^2}=0$

(iv) State and prove Euler's theorem for a function of two variables

Q3 (a) State and prove necessary condition for convergence of positive term series and show that converse is not true (5)

OR

(a) Determine the values of x for which series whose nth term $u_n = \frac{x^{2n-2}}{(n+1)\sqrt{n}}$ converges or diverges

(b) Attempt any two (10)

(i) Discuss the convergence of the series $\frac{1^2}{3} + \frac{2^2}{3^2} + \frac{3^2}{3^3} + \frac{4^2}{3^4} +$ ……

(ii) Show that the sequence $u_n = 1/(1+n^2)$ is a monotonic sequence. Also state it's type

(iii)Prove that the series $\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \frac{1}{5.6} + \cdots$ converges

(iv)Test the convergence of the series whose nth term $u_n = \frac{\sqrt{n+1}-1}{(n+2)^3-1}$

Q4 (a) Change the order of integration $\int_0^{13cos\alpha} \int_{xtan\alpha}^{\sqrt{169-y^2}} f(x,y)dydx$ (5)

OR

(a) Evaluate $\iint ydydx$ over the region bounded by $y = x^2$ and $x + y = 2$

(b) Attempt any two (10)

(i) Find the value of the double integral $\int_2^3 \int_1^2 (x^2 + xy)dydx$

(ii) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} f(x,y)\, dydx$

(iii) Describe the region of integration and evaluate $\int_0^2 \int_0^4 (x^2 + y^2)\, dxdy$

(iv) Evaluate $\iint (x^2 + y^2)dxdy$ over the region S enclosed by x-axis and the lines y = x and x + y =2

Q5 (a) Show that $\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-(y)^{\frac{1}{n}}}dy$ (5)

OR

(a) Show that $\int_0^\infty e^{-h^2x^2}dx = \frac{\sqrt{\pi}}{2h}$

(b) Attempt any two (10)

(i) Show that $\mathcal{B}(m, n) = \mathcal{B}(m, n+1) + \mathcal{B}(m+1, n)$

(ii) Evaluate $\int_0^\infty \frac{x^{\frac{3}{2}}}{(1+x)^5}dx$

(iii) Prove that $\int_0^1 x^{n-1}\left(log\frac{1}{x}\right)^{m-1}dx = \frac{\Gamma(m)}{n^m}$

(iv) Prove that $\mathcal{B}(m, n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}\, dy$