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DF-3018

B. Sc. (Sem. III) Examination

April/May - 2016

MCS-302 : Discrete Mathematics - I

(New Course)

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. Sc. (Sem. III)**

Name of the Subject :
☛ **MCS-302 : Discrete Mathematics - I** *(New Course)*

☛ Subject Code No. : 3 0 1 8 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) First question is compulsory.
(3) Figures to the right indicate full marks of corresponding question.
(4) Follow usual notations.

Q – 1] Answer any **five** of the following questions: **[10]**

1] Define characteristic function.

2] If f: R→R is defined by $f(x) = x^2 + 1$ then find $f^{-1}(-8)$ and $f^{-1}(17)$.

3] Find GCD and LCM of a = -12 and b = 30.

4] For each pair of integers a and b, find integers q and r such that $a = b\,q + r$ and $0 \le r < b$.

(i) a = -111, b = 11 (ii) a = 45, b = 6

5] Identify whether the given recurrence relations are linear homogeneous equations of constant coefficients, if relation is linear homogeneous then find its degree.

(i) $a_n = 2n\, a_{n-1}$ (ii) $a_n = 3\, a_{n-1} - 2\, a_{n-2}$

6] Find the generating function of the numeric function $a_r = 2^r;\ r \ge 0$

7] Define subgroup of a group.

8] Define odd permutation and show that $\begin{pmatrix} 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 3 \end{pmatrix}$ is odd.

Q – 2]

(a) Prove that if a and b are two positive integers then $gcd(a,b) \times lcm(a,b) = ab$. **[05]**

OR

(a) Prove that if gcd (a, b) = 1, and gcd (a, c) = 1 then gcd (a, bc) = 1. **[05]**

(b) Answer **any two** of the following: **[10]**

1] Use prime factorization to find the gcd and lcm of

(i) 143, 227 (ii) 7469, 2464

2] Prove that the square of an integer is either of the form 4k or 4k+1.

3] Use Euclidean algorithm to find the gcd of each pair of integers

(i) 414, 662 (ii) 272, 1479

4] Determine gcd (a, b). Find p and q such that gcd (a, b) = p a + q b where a = 90 and b = 252.

Q – 3]

(a) Find the generating function for a sequence $\{a_k\}$ if $a_k = 2 + 3k$. **[05]**

OR

(a) Write short notes on Fibonacci sequence. **[05]**

(b) Answer **any two** of the following: **[10]**

1] Use generating function to solve $a_{n+2} - 2\, a_{n+1} + a_n = 2^n$; $a_0 = 2$, $a_1 = 1$.

2] Solve the recurrence relation $a_n = a_{n-1} + 2\, a_{n-2}$; $n \geq 2$, $a_0 = 2$, $a_1 = 7$.

3] Solve the difference equation $a_r = a_{r-1} + 6\, a_{r-2}$ where $r \geq 2$ using generating function.

4] Find the generating function of the following sequences:

(i) 0, 0, 1, 1, 1, 1, 1 ...

(ii) 0, 1, -2, 4, -8 ...

Q – 4]

(a) If f: A→B and g: B→C are functions and if f and g are bijections then prove that gof is also a bijection. **[05]**

OR

(a) Let f: A→B and g: B→C and h: C→D then prove that h o (g o f) = (h o g) o f **[05]**

(b) Answer **any two** of the following: **[10]**

1] Check whether the following functions are bijection or not?

(i) $f: R \times R \to R \times R$ defined by $f(x, y) = (x + y, 2x - y)$

(ii) $f: Z \times Z \to Z \times Z$ defined by $f(x, y) = (x + y, x - y)$

2] Let $f: R \times R \to R \times R$ be defined by $f(x, y) = (x + 2y, y - x)$, $g: R \times R \to R \times R$ be defined by $g(x) = (3x, x^2)$. Let $h: R \times R \to R$ be defined by $h(x, y) = x + 2y$. Find f o g, h o f and h o (f o g).

3] Show that function $f(x, y) = x + y$ is a primitive recursive function. Hence compute the value of f (2, 4).

4] Let A = B = C = R and consider f from A to B and g from B to C defined by f (a) = 2a + 1, g (b) = b/3, verify $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

Q – 5]

(a) For a group (G, *), prove that (i) $(a^{-1})^{-1} = a$ (ii) $(a\,b)^{-1} = b^{-1}\,a^{-1}$ **[05]**

OR

(a) Prove that (i) The left identity is also the right identity and **[05]**

(ii) The left inverse is also the right inverse.

(b) Answer **any two** of the following: **[10]**

1] Show that if every element in a group is its own inverse then the group must be abelian.

2] Let R^+ be the multiplicative group of all positive real numbers. Define $f: R^+ \to R^+$ by $f(x) = x^2$ for all $x \in R^+$. Show that f is an automorphism of R^+.

3] Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 1 & 2 \end{pmatrix}$ and

$D = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$. Find AB, BA, A^2, CB and D^{-1}

4] Define a cyclic group. Show that the set $\{1, \omega, \omega^2\}$ is a cyclic group of order 3 with generators ω and ω^2 with respect to multiplication, ω being the cube root of unity.
