DF-3019

Second Year B. Sc. (Sem. III) Examination

March / April - 2016

MCS-303 : Differential Equations - I

(Maths for Comp. Science)

(New Course)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **S.Y. B.SC. (SEM. 3)**

Name of the Subject :
☛ **MCS-303 : DIFFERENTIAL EQUATIONS - 1**

☛ Subject Code No. : **3 0 1 9** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicates full marks.

1 Answer the following questions : (Any Five) **10**

(1) Find complimentary function of $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-54y=0$.

(2) Obtain the general solution of $(\theta^2-1)y=\sin x$.

(3) Convert $x^2\frac{dy}{dx}-2y=x$ into linear differential equation with constant coefficients.

(4) Define : Inverse Laplace Transform.

(5) Find the Laplace transform of the function $F(t)=1$.

(6) Evaluate : $\int_0^\infty t\,e^{-3t}\sin t\,dt$.

(7) If $f(p)=\frac{1}{p}$, then find $L^{-1}\{f(p)\}$.

(8) Show that $\int_0^\infty \frac{\sin t}{t}dt=\frac{\pi}{2}$

2 (a) Describe the method of finding the particular integral of $f(\theta)y = \cos ax$; where $\theta \equiv \frac{d}{dx}$ and $\phi(-a^2) \neq 0$. **5**

OR

(a) Prove that $\frac{1}{f(\theta)} e^{ax} V = e^{ax} \frac{1}{f(\theta + a)} V$; where V is a function of x. **5**

(b) Solve any two from the following : **10**

(1) $(\theta^3 + 1)y = \sin 3x - \cos^2 \frac{1}{2}x$

(2) $(\theta^2 - 4\theta + 3)y = e^x \cos 2x + \cos 3x$

(3) $(\theta^3 + 2\theta^2 + \theta)y = e^{2x} + x^2 + x$

(4) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = x^2 + 4x$

3 (a) Describe the method of finding the general solution of **5**

$$(ax+b)^n \frac{d^n y}{dx^n} + P_1 (ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + ... + P_n y = X(x)$$

OR

(a) Describe the method of finding the general solution of **5**

$$x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + ... + P_n y = X(x).$$

(b) Solve any two from the following : **10**

(1) $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

(2) $(5+2x)^2 \frac{d^2 y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 0$

(3) $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = x^2 . \log x$

(4) $\left(x+a^2\right)\frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$

4 (a) State and prove the second shifting theorem of Laplace transform. **5**

OR

(a) If $L\{F(t)\} = f(p)$, them prove that $L\{F(at)\} = \frac{1}{a} f\left(\frac{p}{a}\right)$. **5**

(b) Find Laplace transform of any two of the following functions : **10**

(1) $f(t) = \begin{cases} 1 \,; 0 < t < 2 \\ t \,; t > 2 \end{cases}$

(2) $e^{-t}\left(3\sin 2t - 5\cosh 2t\right)$

(3) $F(t) = \begin{cases} \sin\left(t - \frac{2\pi}{3}\right); t > \frac{2\pi}{3} \\ 0 \qquad ; t < \frac{2\pi}{3} \end{cases}$

(4) $e^{-4t} \cos h\ 2t$

5 (a) Prove that the inverse Laplace transform possesses the linearity property. **5**

OR

(a) Find the general solution of the differential equation **5**

$\frac{d^2y}{dt^2} + k^2 y = f(t)$ in terms of the constant k and the function $F(t)$.

(b) Answer any two from the following : **10**

(1) Evaluate : $L^{-1}\left\{\frac{3p-2}{p^2-4p+20}\right\}$

(2) Prove that $L^{-1}\left\{\frac{1}{p}\cos\frac{1}{p}\right\}=1-\frac{t^2}{(2!)^2}+\frac{t^4}{(4!)^2}-\frac{t^6}{(6!)^2}+\ldots$

(3) Solve : $\frac{d^2y}{dt^2}-t\frac{dy}{dt}+y=1$; Where $y(0)=1,\ y'(0)=2$.

(4) Find $L^{-1}\left\{\frac{6}{2p-3}-\frac{3+4p}{9p^2-16}+\frac{8-6p}{16p^2+9}\right\}$.