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DG-3152

Third Year B. Sc. (Sem. V) Examination

March/April – 2016

Mathematics - MTH-502

(Linear Algebra - I)

Time : Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **T.Y. B.Sc. (SEM. 5)**

Name of the Subject :
☛ **MATHEMATICS - MTH-502**

☛ Subject Code No. : **3 1 5 2** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All the questions are compulsory.
(3) Figures to the right indicate the marks of the questions.
(4) Follow usual notations.

1. Answer the following as directed : (Any **FIVE**) **(10)**

(1) In a vector space V, prove that $\alpha \cdot \theta = \theta$, for any scalar α.

(2) Give the reason why the subset S = $\{ (x, y, z) \in V_3 \mid x.y = 0\}$ of V_3 is not subspaces of V_3 .

(3) Give an example of two distinct subspaces U and W of a vector space V such that U ∪ W is a subspace of V.

(4) Does (3,7,0) belong to the span of the set S = { (1,2,0), (0,1,0) } of vectors in V_3 ?

(5) Is the set S = { (0,1,0,1), (1,0,0,1), (1,1,0,1) } of vectors in V_4 LI ? Justify your answer.

(6) Are the vectors (1,0,1), (1,1,0), (1,-1,1) coplanar in V_3 ? Justify your answer.

(7) Find the co-ordinate vector of (2,0,-1) relative to the ordered basis B = {(2,1,0), (2,1,1), (2,2,1) } for V_3 .

(8) Let U and W be subspaces of a finite dimensional vector space V. State the formula for dim (U + W) and hence obtain dim (U ⊕ W).

2. Attempt any **TWO** : **(10)**

(1) Prove that $< R^+, +, . >$ is a real vector space ; where + and . are defined as : $u + v = uv$, $\forall u, v \in R^+$ and $\alpha . u = u^\alpha, \alpha \in R, \forall u \in R^+$ & $\forall \alpha \in R$.

(2) In a vector space V, prove that $\alpha . u = \theta$ if and only if either $\alpha = 0$ or $u = \theta$.

(3) State the necessary and sufficient conditions for a non- empty subset S of a vector space V to be a subspace of V. Using it prove that if S is a subspace of a vector space V, then

$$\alpha . u + \beta . v \in S, \text{ for all } u, v \in S \text{ \& scalars } \alpha, \beta.$$

(4) Prove that the set $S = \{ (x, y) \in V_2 \mid a.x+b.y = 0 \}$ is a subspaces of V_2 .

3. Attempt any **TWO** : **(10)**

(1) Let S be a non – empty subset of a vector space V. Then prove that the subspace [S] is the smallest subspace of V containing S.

(2) If $v_1, v_2, \ldots, v_n$ are n vectors in a vector space, then prove that :

(i) $[\alpha_1 . v_1, \alpha_2 . v_2, \ldots, \alpha_n . v_n] = [v_1, v_2, \ldots, v_n]$, where each $\alpha_i \neq 0$;

(ii) $[v_1 - v_2, v_1 + v_2] = [v_1, v_2]$.

(3) Find the span of the set s = { (1,2,1), (1,1,-1), (4,5,-2) }. Is (2,-1,-8) in [S] ?

(4) Define the direct sum. Let U and W be subspaces of a vector space V and $Z = U + W$. If $Z = U \oplus W$, then prove that every vector z in Z can be uniquely expressed as : $z = u + w$; $u \in U, w \in W$.

4. Attempt any **TWO** : **(10)**

(1) Define an LI set. Prove that any set of vectors in a vector space containing the zero vector is LD.

(2) In a vector space V, if the set $\{ v_{1,} v_2, \dots, v_n \}$ is LI and $v \notin [v_{1,} v_2, \dots, v_n]$, then prove that the set $\{ v , v_{1,}, v_2, \dots, v_n \}$ is LI.

(3) Define an LD set. In a vector space V, suppose $\{ v_{1,} v_2, \dots, v_n \}$ an ordered set of vectors with $v_1 \neq \theta$. If one of the vectors $v_2, v_3 \dots, v_n$; say v_k ; belongs to the span of its preceding vectors $v_{1,} v_2, \dots, v_{k-1}$, then prove that the set $\{ v_{1,} v_2, \dots, v_n \}$ is LD.

(4) Check the linear dependence of the following sets :

(i) S = { (1,0,1), (1,1,0), (1,1,-1)} of V_3 ,

(ii) T = { (1,1,0,0), (0,0,1,1), (1,1,1,1) } of V_4 .

5 Attempt any **TWO** : **(10)**

(1) Prove that in an n-dimensional vector space V, any set of n LI vectors is a basis for V.

(2) Define a finite dimensional vector space. Let B = $\{ v_{1,} v_2, \dots, v_n \}$ generate a vector space V. If B is LI, then prove that any vector v in V has the unique expression in terms of $v_{1,} v_2, \dots, v_n$.

(3) Prove that the set B = { (1,-1,2,0), (1,1,2,0), (3,0,0,1), (2,1,-1,0) } is a basis V_4 .

(4) Extend an LI set { (1,0,1,0), (0,-1,1,0) } of vectors in V_4 to a basis for V_4 .