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DG-3153

B. Sc. (Sem. V) (Mathematics) Examination

March/April - 2016

MTH-503 : Real Analysis - I

(New Course)

Time : 2 Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. Sc. (Sem. V) (Mathematics)**

Name of the Subject :

☛ **MTH-503 : Real Analysis - I (New Course)**

☛ Subject Code No. : 3 1 5 3 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) All questions carry equal marks and are compulsory.

(3) Follow usual notations.

Q-1 Attempt any Five; [10]

(1) Define lower bound of a set and find g.l.b for the set $\{\frac{1}{n}\}_{n=1}^{\infty}$.

(2) The sequence $\{\sqrt{n+1}-\sqrt{n}\}_{n=1}^{\infty}$ is convergent or divergent ?Why ?

(3) If S=$\{S_n\}_{n=1}^{\infty} = \{2n-1\}_{n=1}^{\infty}$ } and if N=$\{n_i\}_{i=1}^{\infty} = \{i^2\}_{n=1}^{\infty}$ then find S_7 *and* S_{n_3}.

(4) Show that the sequence$\{|S_n|\}_{n=1}^{\infty}$ converges to 0 then $\{S_n\}_{n=1}^{\infty}$ converges to 0.

(5) Find N∈ I such that $|\frac{2n}{n+3}-2|<\frac{1}{5}$ for n≥ N .

(6) Find limit superior and limit inferior for $\{(1+\frac{1}{n})cosn\pi\}_{n=1}^{\infty}$.

(7) The sequence $\{e^n\}_{n=1}^{\infty}$ convergent or divergent? Why?

(8) Define Cauchy sequence with an illustration .

Q-2 Attempt any Two: [10]

(a) Prove that the countable union of countable sets is countable.

(b) Define a countable set and prove that if B is an infinite subset of a countable set A, then B is also countable .

(c) Prove that the set of all integers is countable

(d) Show that if A and B are countable sets then A×B is also countable .

Q-3 Attempt any Two: [10]

(a) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent then prove that its limit is unique .

(b) If L∈ R, M∈R and L≤M+ε for all $\varepsilon > 0$, then prove that L≤M.

(c) Let $\{S_n\}_{n=1}^{\infty}$ be the sequence defined by

$S_1 = 1$
$S_2 = 1$
$s_{n+1} = s_n + s_{n-1}$ (n=3,4,5......)
$then\ find\ s_8$.

(d) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers with $S_n \geq 0$ for every n and if $\lim_{n\to\infty} s_n = L$ then prove that L≥0.

Q−4 Attempt any Two : [10]

(a) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded .

(b) Prove that a non increasing sequence which is bounded below is convergent.

(c) For n∈ I; $Let\ S_n \frac{2.4.6.....2n}{1.3.5.....(2n-1)} \cdot \frac{1}{n^2}$ Then prove that$\{S_n\}_{n=1}^{\infty}$ is non increasing .

(d) If $\{S_n\}_{n=1}^{\infty}$is a sequence of real numbers, if $\lim_{n\to\infty} s_n = L$ and if c∈R then prove that $\lim_{n\to\infty} cs_n = cL$.

Q − 5 Attempt any Two : [10]

(a) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges , then prove that $\{S_n\}_{n=1}^{\infty}$ is cauchy sequence

(b) Let $\{S_n\}_{n=1}^{\infty}$ diverges to infinity and if $\{t_n\}_{n=1}^{\infty}$ converges then prove that $\{S_n + t_n\}_{n=1}^{\infty}$ diverges to infinity .

(c) If $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded.

(d) Let $\{S_n\}_{n=1}^{\infty}$ diverges to minus infinity and if $\{t_n\}_{n=1}^{\infty}$bounded then prove that $\{S_n + t_n\}_{n=1}^{\infty}$ diverges to minus infinity .
