DG-3154

B. Sc. (Mathematics) (Sem. V) Examination

March/April – 2016

MTH-504 : Real Analysis-II

Time : Hours] [Total Marks :

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.
Name of the Examination :
☛ B. Sc. (Mathematics) (Sem. V)
Name of the Subject :
☛ MTH-504 : REAL ANALYSIS-II
☛ Subject Code No. : 3 1 5 4 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Digits to the right of each question indicate its marks.

(4) Follow usual Symbols.

Q.1 Answer any FIVE from the following. [10]

(1) If $|x-2|<1$, then prove that $|x^2-4|<5$.

(2) If both f and g are continuous at a then show that $\min.\langle f,g\rangle$ is also continuous at a.

(3) If f is real function then define $\lim_{x\to\infty} f(x)=L$.

(4) Define a convergent sequence in a metric space with an illustration.

(5) Define an open ball in metric space. What is $B[a;2]$ in R_d for $a\in R_d$?

(6) Give definition of a metric for a non-empty set M.

(7) Check validity of statement :"Every Cauchy sequence is convergent."

(8) Show that if $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence in R_d then there exists $N\in I$ such that $x_N=x_{N+1}=x_{N+2}=\ldots\ldots$.

Q.2 Answer any TWO from the following. [10]

(a) For $x,y\in R$, define $\sigma(x,y)=|x-y|$. Show that σ is a metric for the set of real numbers, hence show that 2σ is also a metric.

(b) If f and g are the real functions with $\lim_{x\to a} f(x)=L$ and $\lim_{x\to a} g(x)=M$. Then prove that $\lim_{x\to a}[f(x)+g(x)]=L+M$.

(c) (i) If f is continuous at $a\in R$, then prove that $|f|$ is also continuous at $a\in R$.

(ii) Show that if ρ and σ are both metrics for a set M then so $\rho+\sigma$.

(d) If f and g are the real valued functions, if f is continuous at a, and if g is continuous at $f(a)$ then prove that $g \circ f$ is continuous at a.

Q.3 Answer any TWO from the following. [10]

(a) Let $\langle M, \rho \rangle$ be a metric space and if $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of points of M then prove that $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

(b) Show that a sequence of points in any metric space cannot converge to two distinct limits.

(c) Let $\langle M, \rho \rangle$ be a metric space and let a be point in M. Let f and g be real valued functions whose domains are subsets of M. If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = N$. Then prove that $\lim_{x \to a} [f(x) - g(x)] = L - N$.

(d) If σ and ρ are metrics for M and if there exist $k > 1$ such that $\frac{1}{k}\sigma(x,y) \le \rho(x,y) \le k\sigma(x,y)$;$\forall x, y \in M$. Then prove that σ and ρ are equivalent.

Q.4 Answer any TWO from the following. [10]

(a) If f and g are continuous functions from a metric space M_1 into a metric space M_2 then prove that $f \pm g, f \cdot g$ are also continuous on M_1.

(b) Let M be a metric space and suppose $f : M \to R_d$. Show that if f is continuous at $a \in M$ then $\{a\}$ is an open ball in M.

(c) Prove that the function f is continuous at $a \in R^1$ if and only if for given $\varepsilon > 0$ $B[f(a);\varepsilon]$ about $f(a)$ contains an open ball $B[a;\delta]$ about a.

(d) Prove that real function f is continuous at $a \in R$ if and only if $\lim_{n \to \infty} x_n = a \Rightarrow \lim_{n \to \infty} f(x_n) = f(a)$.

Q.5 Answer any TWO from the following. [10]

(a) Let $\langle M_1, \rho_1 \rangle$ and $\langle M_2, \rho_2 \rangle$ be two metric spaces. Let $f : M_1 \to M_2$ then prove that f is continuous on M_1 if and only if whenever G is open in M_2 then $f^{-1}(G)$ is open in M_1.

(b) Prove that every open ball in a metric space is an open set.

(c) If A and B are open subsets of R^1 then prove that $A \times B$ is an open subset in R^2.

(d) If $G_1, G_2, \ldots, G_n$ are open subsets of a metric space M then prove that the set $\bigcap_{i=1}^{n} G_i$ is also an open set in M.
