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DG-3156

B. Sc. (Sem. V) Examination

March/April – 2016

Mathematics : Paper : MTH - 506

(Number Theory - I)

Time : 2 Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. Sc. (Sem. V)**

Name of the Subject :

☛ **Mathematics : Paper : MTH - 506**

☛ Subject Code No. : **3 1 5 6** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) First quesiton is compulsory.

(3) Figures to the right indicate marks of the question.

(4) Follow usual notations.

(5) Use of non-programmable scientific calculator is allowed.

Que 1: Answer any **FIVE** of the following: **(10)**

1) Prove that square of any integer is either of the form $3k$ or $3k + 1$.
2) If $a|c$ and $b|c$, with $\gcd(a, b) = 1$, then prove that $ab \mid c$.
3) Prove that any integer of the form $n^4 + 4$; $(n > 1)$ is always composite.
4) State the Fundamental Theorem of Arithmetic.
5) Check whether 1009 is a prime or composite?
6) Give an example of CRS modulo 8.
7) If $N = 123x7$ is divisible by 9, then find x.
8) Find the remainder when $2^{20} - 1$ is divided by 41.

Que 2 : Answer any **TWO** of the following: **(10)**

(1) Given integers a and b, with $b > 0$. Show that there exist integers q and r such that $a = bq + r$; $0 \leq r < b$.

(2) Given integers a and b, not both zero. Prove that there exist integers x and y such that $\gcd(a, b) = ax + by$.

(3) Find the integers x and y such that $\gcd(272, 1479) = 272x + 1479y$.

(4) For $n \geq 1$, prove that $6|n(n + 1)(2n + 1)$.

Que 3 : Answer any **TWO** of the following: **(10)**

(1) Show that the linear Diophantine equation $ax + by = c$ has a solution if and only if $d|c$, where $d = \gcd(a, b)$. Also when $x = x_0, y = y_0$ is a particular solution , then find all other solutions.

(2) Prove that $\sqrt{2}$ is irrational.

(3) Determine all the positive solutions of the Diophantine equation $172x + 20y = 1000$.

(4) If $p \geq 5$ is a prime then show that $p^2 + 2$ is composite.

Que 4 : Answer any **TWO** of the following: **(10)**

(1) If p_n is the n^{th} prime number, then prove that $p_n \leq 2^{2^{n-1}}$.

(2) If $a \equiv b (mod\ n)$, then prove that $\gcd(a, n) = \gcd(b, n)$.

(3) Prove that the integer $111^{333} + 333^{111}$ is divisible by 7.

(4) Prove that the congruence relation modulo n, of integers, is an equivalence relation.

Que 5 : Answer any **TWO** of the following: **(10)**

(1) Let $N = a_m 8^m + a_{m-1} 8^{m-1} + a_{m-2} 8^{m-2} + \cdots + a_1 8^1 + a_0$, be the representation of the positive integer N, with $0 \leq a_k \leq 7$, and let $T = a_0 + a_1 + a_2 + \cdots + a_{m-1} + a_m$. Then prove that $7|N$ if and only if $7|T$.

(2) Let $N = a_m 10^m + a_{m-1} 10^{m-1} + a_{m-2} 10^{m-2} + \cdots + a_1 10^1 + a_0$ be the representation of the positive integer N, with $0 \leq a_k \leq 9$, and let $T = a_0 - a_1 + a_2 + \cdots + (-1)^m a_m$. Then prove that $11|N$ if and only if $11|T$.

(3) Find the last two digits of 9^{9^9}.

(4) Working modulo 9 or 11, find the missing digit x in the following calculation: $51840 \times 273581 = 1418243x040$.
