* D M M - 3 0 9 6 *

DMM-3096

B. Sc. (Sem. IV) Examination

March/April – 2016

Mathematics : Paper - MTH-401

(Advanced Calculus-II)

Time : **2** Hours] [Total Marks : **50**

Instructions : (1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. Sc. (Sem. IV)**

Name of the Subject :

☛ **Mathematics : Paper - MTH-401**

☛ Subject Code No. : **3 0 9 6** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of corresponding question.

(4) Follow usual notations.

Q:1 Answer any **FIVE** of the following questions : **(10)**

(1) Define Gamma integral. Find the value of $\Gamma(1)$.

(2) State the first shifting theorem for inverse Laplace transform.

(3) Find $L\{sinhat\}$.

(4) Prove that $f(x,y)=x^4+y^4$ is minimum at (0, 0).

(5) Find Laplace transform of t^2e^{-4t}.

(6) Evaluate: $\int_0^1 x^2(1-x)^4\,dx$.

(7) State the necessary and sufficient conditions for an extreme point of a bivariate function.

(8) Evaluate: $L^{-1}\left[\frac{1}{p^2-8p+18}\right]$.

Q:2 Answer any **TWO** of the following questions : **(10)**

(a) Find the interior point of the triangle, such that the sum of the squares of its distances from the vertices is minimum.

(b) Show that $f(x,y)=x^3+y^3-3x-12y+20$ attains minimum value 2 at $(1,2)$.

(c) Discuss about the extreme points of $u = x^3 + y^3 - 3axy$.

(d) Find the extreme values of $u = x^3 + y^3 - xy$.

Q:3 Answer any **TWO** of the following questions : **(10)**

(a) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(b) Evaluate: $\int_0^4 \frac{x^2}{\sqrt{4-x}} dx$.

(c) Show that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} \int_0^{\frac{\pi}{2}} \sqrt{\sin\theta} d\theta = \pi$.

(d) State and prove the relation between Beta and Gamma function.

Q:4 Answer any **TWO** of the following questions : **(10)**

(a) Show that the Laplace transformation is linear. Also obtain $L[\sin at]$.

(b) Find $L\{\sin\sqrt{t}\}$.

(c) Evaluate: $\int_0^{\infty} \frac{e^{-t} - e^{-3t}}{t}$.

(d) State and prove the change of scale property for Laplace transform.

Q:5 Answer any **TWO** of the following questions : **(10)**

(a) State and prove change of scale property for inverse Laplace transform.

(b) Show that $L^{-1}\left[\frac{3p-2}{p^2-4p+20}\right] = e^{2t}(3\cos 4t + \sin 4t)$.

(c) Find $L^{-1}\left\{\frac{6}{2p-3} - \frac{3+4p}{9p^2-16} + \frac{8-6p}{16p^2+9}\right\}$.

(d) Evaluate: $L^{-1}\left[\frac{p+2}{p^2-2p+5}\right]$.