DMM-3099

Second Year B. Sc. (Sem. IV) Examination

March / April - 2016

MCS-401 : Mathematics for Computer Science (Graph Theory)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **Second Year B. Sc. (Sem. IV)**

Name of the Subject :
☛ MCS-401 : Mathematics for Computer Science (Graph Theory)

☛ Subject Code No. : 3 0 9 9 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All the question are compulsory.

(3) Digits shown in the right hand side indicate full marks of the question.

(4) Symbols have their usual meaning.

1 Attempt any five from the following : **10**

(i) Define simple graph, pseudo graph and multi graph and give an example of each.

(ii) Draw digraph $G(V, R), V = \{1, 2, 3, 4\}$ and

$R = \{(x, y) \mid |x - y| \leq 1\}$, state the type of relations it is.

(iii) Is there exists a simple graph with the degree sequence (1,1,3,4,6,7) ? If exists draw the graph otherwise explain.

(iv) Draw $K_{2,5}$ graph, name the graph. Find the number of edges also.

(v) A binary tree has 17 nodes find the number of edges, interior nodes and leaves.

(vi) Determine the number of edges in a graph with 6 nodes, 2 nodes are of degree 3 and 4 nodes are of degree 2.

(vii) Define walk and path give illustration.

2 (a) Show that $|E| \leq 3|V| - 6$ is a necessary but not a sufficient condition for a simple connected graph to be planar. Give counter example in support. **5**

OR

(a) Is it possible to have a graph with 17 edges such that 3 of its nodes have degree 3 and the rest of the nodes have degree 5? What about if 4 nodes are of degree 3 and the rest of degree 5? Draw the graph in case it exists.

(b) Attempt any two : **10**

(i) Define 3-regular graph. If is a 3 regular graph with n vertices, find the sum of the degrees of all vertices. Show that in such a graph n must be even.

(ii) Define complement of a graph. Let G be a simple graph with 9 vertices and 12 edges. Find the number of edges in complement of G.

(iii) Let G be a simple graph with 6 vertices and 11 edges. Show that the graph G is connected.

(iv) Explain seating problem.

3 (a) There are 102 students in a class. Number of computers in lab is 35. Each student will be assigned to use 1 of 35 computers, and each of the 35 computers will be used by exactly 1 or 3 students. Either find the graph that models the above situation or show that none exist. **5**

OR

(a) For each of the following degree sequence, find if there exists a graph. In case graph exists draw it or explain why no graph exists.

(i) 5,5,4,3,2,1

(ii) 3,3,3,3,2

(b) Attempt any **two** : **10**

(i) If $G=(V,E)$ is a (p,q) graph then show that $\delta \le \frac{2q}{p} \le \Delta$, where δ is the minimum of all degrees of the vertices of the graph and Δ is the maximum of all degrees of the vertices of the graph.

(ii) Find the number of edges in planar graph with 5 nodes and 7 regions. Draw such graph.

(iii) Draw 2-regular graph on 6 nodes, K_5, 3-regular graph on 4 nodes, K_4 and $K_{2,3}$.

(iv) Define : Walk, Trial, Path, circuit, cycle together with example.

4 (a) Define spanning tree and write step by step procedure of Prim's algorithm to get minimal spanning tree. Illustrate the method by example. **5**

OR

(a) Define binary tree and complete binary tree. Find the number of leaves and internal nodes in complete binary tree with 25 vertices.

(b) Attempt any **two** : **10**

(i) Show that minimum height of complete binary tree is smallest integer greater than or equal to $\log_2(n+1)-1$.

(ii) Define binary and complete binary tree. Draw all distinct complete binary tree having 7 vertices and height 3.Find the sum of path lengths from root to terminal vertices in each case.

(iii) Show that a tree with n vertices has exactly $n-1$ edges.

(iv) Show that the number of leaves on an complete m- ary tree with i interior nodes is $(m-1)i+1$. Find the number of leaves on 5-ary tree with 26 interior nodes.

5 (a) A connected planar graph has 15 edges each node is of degree 3.Find the number of regions in which it will split the plane. **5**

OR

(a) Draw the graph G having 7 edges in which 1 vertex has degree 2 and others have degree 3. Also find complement of *G*.

(b) Attempt any two : **10**

(i) Draw the graphs represented by the following matrices :

$G =$

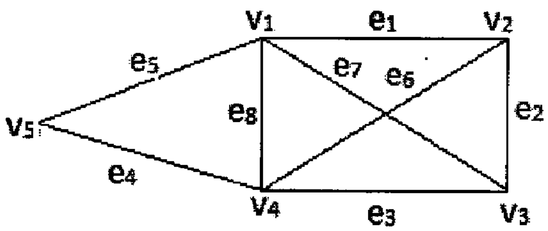
0	1	0	1
1	0	1	0
0	1	0	1
1	0	1	0
0	0	0	1

$H =$

1	1	1	0
1	0	1	0
0	0	1	1
0	1	1	1
1	0	1	1

Also find $G \cup H$ and $G \cap H$.

(ii) Consider the graph given below :



Find

(1) Path from v_1 to v_2 of length 4.

(2) Simple path from v_1 to v_5 of length 4.

(3) Does there exists cycle with end nodes v_5, find the length of it.

(iii) Let *G* is a simple graph with 40 edges and its complement has 26 edges. Find the number of vertices in *G*.

(iv) Does there exists a 3-regular graph on 7 vertices? What about a 4-regular graph on 7 vertices. If so draw graph.