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DMM-3100

B. Sc. (Sem. - IV) Examination

March/April – 2016

Mathematics : Paper - MCS - 402

(Discrete Mathematics-II)

Time : 3 Hours] [Total Marks : **70**

Instructions : (1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. SC. (SEM. - 4)**

Name of the Subject :
☛ **MATHEMATICS : PAPER - MCS - 402**

☛ Subject Code No. : **3 1 0 0** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) First question is compulsory.
(3) Figures to the right indicate full marks of the question.
(4) Follow usual notations.
(5) Use of non-programmable scientific calcualtor is allowed.

Q-1] Answer the following: **[10]**

1] Define linear congruence. How many solutions are there to the following congruence?

$15x \equiv 24 \pmod{35}$

2] Define Euler phi function (totient function). Find φ (15).

3] Draw Hasse diagram for the power set of A = {a, b, c}.

4] Prove that 1 is the only complement of 0.

5] Define Boolean homomorphism.

Q-2] (a) State and prove Fermat's Little theorem. **[05]**

OR

(a) Let $N = a_k 10^k + a_{k-1} 10^{k-1} + ...+ a_2 10^2 + a_1 10 + a_0$, where $a_0, a_1,...,a_k$ are integers such that $a_k \neq 0$ and $0 \le a_i \le 10$ for i = 0, 1,...,k. Let $S = a_0 + a_1 + ...+ a_k$ and $T = a_0 - a_1 + a_2 - ...+ (-1)^k a_k$, then

(i) N is divisible by 9 if and only if S is divisible by 9.

(ii) N is divisible by 11 if and only if T is divisible by 11.

(b) Answer (*any two*) of the following: **[10]**

1] The ISBNs of books are: (i) 81-203-148c-8 (ii) 81-754c-136-4 (iii) 81-21c-2089-2

Find the values of c.

2] Solve $345x \equiv 15 \pmod{912}$.

3] Find remainder when sum 1! + 2! + ... + 99! + 100! is divided by 12.

4] Prove that $41 / 2^{20} - 1$ using theory of congruence.

Q-3] (a) Show that every closed interval of a lattice is a sublattice. **[05]**

OR

(a) Show that De' Morgan's laws hold in a complemented distributive lattice.

(b) Answer (*any two*) of the following: **[10]**

1] Let I^+ be a set of positive integers and D denote the relation of divisor in I^+, such that for any a, b $\in I^+$ and a D b implies a/b. Prove that $\langle I^+, D\rangle$ is a lattice.

2] Show that in $\langle L, \leq\rangle$ if $a \leq b$ and $c \leq d$ then

(i) $a * c \leq b * d$ (ii) $(a * b) \oplus (c * d) \leq (a \oplus c) * (b \oplus d)$

3] Let $< L, \leq >$ be a lattice, for any $a, b \in L$ prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

4] Let $< L, *, \oplus >$ be a lattice for any two elements $a, b \in L$ such that $a \leq b$.

The set $[a, b] = \{x \in L; a \leq x \leq b\}$ then prove that the set $[a, b]$ is a sublattice of L.

Q-4] (a) Show that a lattice homomorphism on a Boolean algebra which preserves 0 and 1 is a Boolean homomorphism. **[05]**

OR

(a) Let S be any non empty set and $\rho(S)$ be its power set, prove that $< \rho(S), \cap, \cup, \sim, 0, 1 >$ is a Boolean algebra.

(b) Answer (*any two*) of the following: **[10]**

1] Find the value of the Boolean expression $f(x_1, x_2, x_3) = \sum 0, 3, 5, 7$. Find f (a, b, 1)

and $(x_1 * x_2) * [(x_1 * x_4) \oplus x_2' \oplus (x_3 * x_1')]$. Find $\alpha\ (x_1, x_2, x_3, x_4) = \alpha\ (a, 1, b, 1)$.

2] Find the value of $(a * b)' \oplus (a \oplus b)'$.

3] Let $< B, *, \oplus, ', 0, 1 >$ be a Boolean algebra and $S \subseteq B$. If S is closed w.r.t. $'$and $\oplus$ then, prove that $< S, *, \oplus, ', 0, 1 >$ is a sub boolean algebra.

4] Obtain the Boolean function $f_{\alpha,\beta}: B^2 \rightarrow B$ associated with the Boolean expression $\alpha\ (x, y) = x \oplus y$.

Q-5] (a) Define Distributive and Modular lattice. Prove that every distributive lattice is a modular lattice. **[05]**

OR

(a) Prove that following expressions are equivalent & find its sum of product canonical form:

(i)$(x \oplus y) * (x' \oplus z) * (y \oplus z)$ (ii)$(x \oplus y) * (x' \oplus z)$

(iii)$(x * z) \oplus (x' * y) \oplus (y * z)$ (iv)$(x * z) \oplus (x' * y)$

(b) Answer (*any two*) of the following: **[10]**

1] Obtain minimal function by Karnaugh map representation of f (a, b, c, d) = $\sum$ (0, 1, 2, 3, 13, 15).

2] Find the sum of product cannonical form of $x_1 \oplus (x_2 * x_3')$.

3] Which of the two lattices $<S_n, D>$ for n = 30 and n = 45 are complemented? Are these lattices distributive?

4] Find the product of sum cannonical form of $(x_1 \oplus x_2)' * x_3$.