DMM-3334

B. Sc. (Sem. IV) Examination

March/April – 2016

Mathematics : Paper - CCM - 402CS

(Discrete Mathematics-I) (Old Course)

Time : 3 Hours] [Total Marks : **70**

Instructions : (1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. Sc. (Sem. IV)**

Name of the Subject :
☛ **Mathematics : Paper - CCM - 402CS (Old)**

☛ Subject Code No. : 3 3 3 4 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) First question is compulsory.
(3) Figures to the right indicate full marks of the question.
(4) Follow usual notations.
(5) Use of non-programmable scientific calcualtor is allowed.

Q-1] Answer the following: **[10]**

1] Define Composition of relations and find R^2 for R = {(1, 1), (2, 1), (3, 2)}.

2] Explain Symmetric relation with illustration.

3] Prove that $(a * b)' \oplus (a \oplus b)' = a' \oplus b'$.

4] Define Partial order relation and totally ordered set.

5] Define Boolean homomorphism.

Q-2] (a) Explain Reflexive, Anti symmetric and Transitive relations. Find each of them for the relation R = {(i, j): |i-j| = 2} where the set is {1, 2, 3, 4, 5, 6}. **[05]**

OR

(a) Explain closure of relations in detail.

(b) Answer (*any two*) of the following: **[10]**

1] Prove that a chain of 3 or more elements is not complemented.

2] If R = {(x, y): x ∈ Z, y ∈ Z, (x-y) is divisible by 6}, then prove that R is an equivalence relation.

3] Prove that the operation of meet and join satisfy absorption laws.

4] If $< L, \leq >$ is a lattice, for any $a, b, c \in L$, show that the following properties hold:

$b \leq c \Rightarrow a * b \leq a * c$ and $a \oplus b \leq a \oplus c$.

Q-3] (a) Show that the operations of meet and join satisfy commutative, associative and idempotent laws. **[05]**

OR

(a) Show that De' Morgan's laws hold in a complemented distributive lattice.

(b) Answer (*any two*) of the following: **[10]**

1] Let S_n be a set of positive divisors of n and D denote the relation of division. Prove that $< S_n$, D> is a lattice for n = 6.

2] Solve the congruences $2x \equiv 4 \pmod 5$ and $x + 1 \equiv 3 \pmod 5$.

3] Let $< L, \leq >$ be a lattice, for any $a, b \in L$ prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

4] Let $< L, *, \oplus >$ be a lattice for any two elements $a, b \in L$ such that $a \leq b$.

The set $[a, b] = \{x \in L; a \leq x \leq b\}$ then prove that the set $[a, b]$ is a sublattice of L.

Q-4] (a) Show that a lattice homomorphism on a Boolean algebra which preserves 0 and 1 is a Boolean homomorphism. **[05]**

OR

(a) Let S be any non empty set and $\rho(S)$ be its power set, prove that $< \rho(S), \cap, \cup, \sim, 0, 1 >$ is a Boolean algebra.

(b) Answer (*any two*) of the following: **[10]**

(1) Find the complement of following Boolean expression :

(i) $xy' + x'z$ **(ii)** $ab' + ac + b'c$

2] Prove that $[a * (b' \oplus c)]' * [b' \oplus (a * c')']' = a * b * c'$.

3] Let $< B, *, \oplus, ', 0, 1 >$ be a Boolean algebra and $S \subseteq B$. If S is closed w.r.t. $'$ and $\oplus$ then, prove that $< S, *, \oplus, ', 0, 1 >$ is a sub boolean algebra.

4] Obtain the Boolean function $f_{\alpha,\beta}: B^2 \rightarrow B$ associated with the Boolean expression $\alpha(x, y) = x * y$.

Q-5] (a) Use K-map representation to find a minimal sum of product canonical form of the function f(a, b, c) = a' b' c' + a' b' c + a b' c' + a b c'. **[05]**

OR

(a) Prove that following expressions are equivalent & find its sum of product canonical form:

(i)$(x \oplus y) * (x' \oplus z) * (y \oplus z)$ (ii)$(x \oplus y) * (x' \oplus z)$

(iii)$(x * z) \oplus (x' * y) \oplus (y * z)$ (iv)$(x * z) \oplus (x' * y)$

(b) Answer (*any two*) of the following: **[10]**

1] Obtain minimal function by Karnaugh map representation of f (a, b, c, d) = ∑ (0, 2, 6, 8, 10, 12, 14, 15).

2] Find the sum of product cannonical form of $x_1 \oplus (x_2 * x_3')$.

3] Find the characteristic number of following symmetric expression:

$(x_1 * x_2' * x_3') \oplus (x_1' * x_2 * x_3') \oplus (x_1' * x_2' * x_3) \oplus (x_1 * x_2 * x_3)$

4] Find the product of sum cannonical form of $x_1 * x_2$.