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DMM-3335

Second Year B. Sc. (Sem. IV) Examination

April / May - 2016

CCM - 403 : Graph Theory

(Mathematics for Computer Science) (Old Course)

Time : Hours] [Total Marks :

Instruction :

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **Second Year B. Sc. (Sem. IV)**

Name of the Subject :
☛ CCM - 403 : Graph Theory *(Old Course)*

☛ Subject Code No. : 3 3 3 5 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

1 Answer any five from the following : **10**

(1) Draw an Euler graph which is not arbitrary traceable.

(2) If $g \leq G$, then show that $G \oplus g = G - g$.

(3) In how many ways a graph containing 5 edges can be decomposed into pairs of subgraphs g_1 and g_2 ?

(4) Define with illustration : Spanning tree.

(5) State the number of Hamiltonian circuits in a complete graph with 9-vertices.

(6) Define :

(i) Self loop

(ii) Degree of vertex.

(7) Draw all binary tree with 6 leaves.

(8) Sketch all different (non isomorphic) simple digraphs with 1, 2 and 3 vertices.

2 (a) Prove that a graph with n vertices, n–1 edges and no circuit is connected. **5**

OR

(a) Prove that a connected graph with n vertices and e edges has e–n+2 regions. **5**

(b) Attempt any two of the following : **10**

(1) Explain : Konigsberg bridge problem.

(2) Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Draw two such graphs, one simple, one not.

(3) Prove that a simple graph with n-vertices and k components has at the most $\frac{1}{2}$ (n–k) (n–k–1) edges.

(4) Define with illustration :

(i) Parallel edges

(ii) Simple graph.

3 (a) Define complete graph and show that a Hamiltonian circuit can be constructed in a complete graph. **5**

OR

(a) Prove that a connected graph has an Euler circuit iff all the vertices of it are of even degree. **5**

(b) Attempt any two : **10**

(1) Prove that a connected graph with exactly 2 nodes of odd degree has an Euler path.

(2) Prove that a connected graph is an Euler graph iff it can be decomposed into circuits.

(3) Draw a graph that has exactly one Euler circuit. Characterize all such graphs. Does there exist an Euler circuit in a graph with a vertex of degree 3 ?

(4) Determine the maximal edge connectivity of a connected graph with 5 nodes and 8 edges. Draw a graph in which the connectivity is met and one in which it is not.

4 (a) Prove that a digraph G is an Euler digraph iff G is connected and is balanced. **5**

OR

(a) Prove that the determinant of every square submatrix of A, the incidence matrix of a digraph, is 1, –1 or 0. **5**

(b) Attempt any **two** : **10**

(1) How many edges are there in a digraph with 5 vertices, each of which has out-degree 2 ? Draw such a digraph.

(2) Sketch all distinct (non isomorphic) orientations of a complete graph of 4 vertices. Characterize each of the resulting digraphs in terms of binary relations.

(3) Define simple, symmetric and balanced digraphs.

(4) Define :

(i) Incidence Matrix

(ii) Adjacency Matrix

(iii) Circuit Matrix.

5 (a) Explain Prim's Algorithm to obtain spanning tree with illustration. **5**

OR

(a) Prove that a tree with n vertices has n–1 edges. **5**

(b) Attempt any **two** : **10**

(1) Prove that every connected graph has at least one spanning tree.

(2) Draw all rooted trees with 5 nodes.

(3) What is the height of a balanced binary tree with 51 vertices ?

(4) Prove that a graph is a tree iff it is minimally connected.