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DMM-3341

B. Sc. (Sem. - IV) Examination

March/April – 2016

Mathematics : CCM-401

(Advanced Calculus-II) (Old)

Time : **2** Hours] [Total Marks : **50**

Instructions : (1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. SC. (SEM. - IV)**

Name of the Subject :

☛ **MATHEMATICS : CCM-401 (OLD)**

☛Subject Code No. : **3 3 4 1** ☛Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of corresponding question.

(4) Follow usual notations.

Q:1 Answer any **FIVE** of the following questions : **(10)**

(1) Find $L\{coshat\}$.

(2) Evaluate: $\int_0^1 x^2(1-x)^4\,dx$.

(3) State the linearity property of inverse Laplace transform.

(4) Check the validity of $\int_1^3\int_0^2 dx\,dy = \int_1^3\int_0^2 dy\,dx$.

(5) Evaluate: $L^{-1}\left[\frac{1}{p^2 - 8p + 19}\right]$.

(6) Evaluate: $\int_0^1\int_0^{\frac{y}{2}} dy\,dx$.

(7) Find $L\{t^4 + 4e^{-3t} - 2\sin 4t\}$.

(8) Show that $(m,n) = \beta(m+1,n) + \beta(m,n+1)$.

Q:2 Answer any **TWO** of the following questions : **(10)**

(a) Find the area between two parabolas $y^2 = 2x$ and $x^2 = 2y$.

(b) Evaluate: $\int_0^1 \int_y^{2-y} \left(x^2+y^2\right) dx\,dy$ after changing the order of integration.

(c) Evaluate: $\int_0^{\pi/2} \int_0^{2a\cos\theta} r^2 \sin\theta \cos\theta \; d\theta \; dr.$

(d) Change the order of integration of the double integral $\int_0^5 \int_{12y/5}^{\sqrt{169-y^2}} f(x,y) \; dy \; dx$.

Q:3 Answer any **TWO** of the following questions : **(10)**

(a) State and prove the relation between Beta and Gamma function.

(b) Evaluate: $\int_0^3 \frac{x^2}{\sqrt{3-x}} \, dx.$

(c) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(d) Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \int_0^{\pi/2} \sqrt{\sin\theta} \; d\theta = \pi$.

Q:4 Answer any **TWO** of the following questions : **(10)**

(a) Show that the Laplace transformation is linear. Also obtain $L[\sinh at]$.

(b) State and prove the change of scale property for Laplace transform.

(c) Find $L\left\{e^t \sin^2 t\right\}$.

(d) Find Laplace transformation of $F(t) = \begin{cases} \cos\left(t-\frac{2}{3}\pi\right), & t > \frac{2}{3}\pi \\ 0, & t < \frac{2}{3}\pi \end{cases}$.

Q:5 Answer any **TWO** of the following questions : **(10)**

(a) State and prove second shifting theorem for inverse Laplace transform.

(b) Find $L^{-1}\left\{\frac{6}{2p-3} - \frac{3+4p}{9p^2-16} + \frac{8-6p}{16p^2+9}\right\}$.

(c) Prove that $L^{-1}\left\{\frac{p}{(p^2+1)^2}\right\} = \left(\frac{t}{2}\right)\sin t \Rightarrow L^{-1}\left\{\frac{72p}{(36p^2+1)^2}\right\} = \left(\frac{t}{6}\right)\sin\left(\frac{t}{6}\right)$.

(d) Show that $L^{-1}\left[\frac{1}{p}\cos\frac{1}{p}\right] = 1 - \frac{t^2}{(2!)^2} + \frac{t^4}{(4!)^2} - \frac{t^6}{(6!)^2} + \ldots$