DMM-3341  
B. Sc. (Sem. - IV) Examination  
March/April – 2016  
Mathematics : CCM-401  
(Advanced Calculus-II) (Old)  

Time : 2 Hours]  
[Total Marks : 50  

Instructions : (1)  

(2) All questions are compulsory.  
(3) Figures to the right indicate marks of corresponding question.  
(4) Follow usual notations.  

Q:1 Answer any FIVE of the following questions :  
(10)  
(1) Find \( L\{\cos\theta\} \).  
(2) Evaluate: \( \int_{0}^{1} x^2 (1-x)^4 \, dx \).  
(3) State the linearity property of inverse Laplace transform.  
(4) Check the validity of \( \int_{0}^{2} \int_{0}^{1} dy \, dx = \int_{1}^{2} \int_{0}^{1} dy \, dx \).  
(5) Evaluate: \( \mathcal{L}^{-1}\left[ \frac{1}{p^2 - 8p + 19} \right] \).  
(6) Evaluate: \( \int_{0}^{\frac{\pi}{2}} dy \).  
(7) Find \( L\{t^4 + 4e^{-3t} - 2\sin 4t\} \).  
(8) Show that \( \beta(m, n) = \beta(m+1, n) + \beta(m, n+1) \).  

Q:2 Answer any TWO of the following questions :  
(10)  
(a) Find the area between two parabolas \( y^2 = 2x \) and \( x^2 = 2y \).  

DMM-3341]  
[Contd...
(b) Evaluate: \[
\int_0^1 \int_y^{2-y} \left(x^2 + y^2\right) \, dx \, dy
\] after changing the order of integration.

(c) Evaluate: \[
\int_0^{\pi/2} \int_0^{2 \arccos \theta} r^2 \sin \theta \cos \theta \, d\theta \, dr.
\]

(d) Change the order of integration of the double integral
\[
\int_0^5 \int_{\sqrt{169-y^2}}^{12y/5} f(x, y) \, dy \, dx.
\]

Q:3 Answer any TWO of the following questions:

(a) State and prove the relation between Beta and Gamma function.

(b) Evaluate: \[
\int_0^3 - \frac{x^2}{\sqrt{3-x}} \, dx.
\]

(c) Prove that \[
\Gamma \left( \frac{3}{2} \right) = \sqrt{\pi}.
\]

(d) Show that \[
\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \int_0^{\pi/2} \sqrt{\sin \theta} \, d\theta = \pi.
\]

Q:4 Answer any TWO of the following questions:

(a) Show that the Laplace transformation is linear. Also obtain \[L[sinh at].\]

(b) State and prove the change of scale property for Laplace transform.

(c) Find \[L\{e^{2t} \sin^2 t\} \].

(d) Find Laplace transformation of \[F(t) = \begin{cases} \cos \left(t - \frac{2}{3} \pi \right), & t > \frac{2}{3} \pi \\ 0, & t < \frac{2}{3} \pi \end{cases}\].

Q:5 Answer any TWO of the following questions:

(a) State and prove second shifting theorem for inverse Laplace transform.

(b) Find \[L^{-1} \left\{ \frac{6}{2p-3} - \frac{3+4p}{9p^2-16} + \frac{8-6p}{16p^2+9} \right\} \].

(c) Prove that \[L^{-1} \left\{ \frac{p}{(p^2+1)^2} \right\} = \left( \frac{t}{2} \right) \sin t \Rightarrow L^{-1} \left\{ \frac{72p}{(36p^2+1)^2} \right\} = \left( \frac{t}{6} \right) \sin \left( \frac{t}{6} \right).\]

(d) Show that \[L^{-1} \left[ \frac{1}{p} \cos \frac{1}{p} \right] = 1 - \frac{t^2}{(2!)^2} + \frac{t^4}{(4!)^2} - \frac{t^6}{(6!)^2} + \ldots.\]