DMM-3343
B. Sc. (Sem. IV) Examination
March/April – 2016
Mathematics : MTH-403
(Old Course) (Numerical Analysis-II)

Time : 2 Hours] [Total Marks : 50

Instructions : (1)

(2) Digits to the right indicates marks of the question.
(3) Follow the usual notations.
(4) Use of Scientific non-programmable calculator is allowed.

Q-1 Answer any five as directed : (10)

(1) Discuss the disadvantage of Lagrange’s Interpolation formula and advantage of Divided Difference Interpolation formula.

(2) If \( f(0) = -1 \) and \( f(3) = 2 \) then obtain the function \( f(x) \) by using Lagrange’s Interpolation formula.

(3) If \( f(x) = \frac{1}{x} \) then find \( [x_0, x_1, x_2] \).

(4) Prove that \( [x_0, x_1] = [x_1, x_0] \)

(5) Construct divided difference table:

\[
\begin{array}{|c|c|c|c|c|}
\hline
x & 0 & -1 & 2 & 4 \\
\hline
f(x) & -2 & -6 & 4 & 10 \\
\hline
\end{array}
\]

(6) Write the formula to obtain the first derivative at \( x = x_0 \) and second derivative at \( x = x_n \)

(7) What is the necessary condition for applying the Simpson rule?

(8) Define Initial value problem.
Q-2 Answer any two as directed:

(1) Derive Newton's Divided Difference Interpolation formula.
(2) Derive Lagrange's Interpolation formula.
(3) Express the rational function \( \frac{3x^2 + x + 1}{x^3 - 6x^2 + 11x - 6} \) as sums of partial fraction.
(4) Given the set of tabulated points \((1, -3), (3, 9), (4, 30)\) and \((6, 132)\) satisfying the function \( y = f(x) \), compute \( f(5) \) using Newton's divided difference interpolation formula.

Q-3 Answer any two as directed:

(1) By using Newton's forward formula obtain the formula for numerical differentiation of first order at \( x = x_0 \).
(2) By using Newton's backward formula obtain the formula for numerical differentiation of second order at \( x = x_n \).
(3) From the following table of values of \( x \) and \( y \) obtain \( \frac{dy}{dx} \) at \( x = 2.2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
<th>2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3.3201</td>
<td>4.0552</td>
<td>4.9530</td>
<td>6.0496</td>
<td>7.3891</td>
<td>9.0250</td>
</tr>
</tbody>
</table>

(4) From the following table of values of \( x \) and \( y \) obtain \( \frac{d^2y}{dx^2} \) at \( x = 1.4 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
<th>2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4.0552</td>
<td>4.9530</td>
<td>6.0496</td>
<td>7.3891</td>
<td>9.0250</td>
</tr>
</tbody>
</table>

Q-4 Answer any two as directed:

(1) State and prove Simpson's 3/8 rule.
(2) State and prove Trapezoidal rule.
(3) Evaluate \( \int_0^1 \frac{1}{1+x} \, dx \) by Trapezoidal rule. (\( h = 0.125 \))
(4) Evaluate \( \int_0^1 \sqrt{1-x^2} \, dx \) by taking \( h = \frac{1}{6} \).

Q-5 Answer any two as directed:

(1) Obtain the numerical solution of initial value problem \( \frac{dy}{dx} = f(x, y) \)
by Taylor's series method where \( y(x_0) = y_0 \)
(2) Obtain the numerical solution of initial value problem \( \frac{dy}{dx} = f(x, y) \)
equation by Picard's method where \( y(x_0) = y_0 \).
(3) The differential equation \( \frac{dy}{dx} = \frac{1}{x^2 + y} \) with \( y(4) = 4 \) obtain \( y(4.1) \) and \( y(4.2) \) by Taylor's series method.
(4) The differential equation \( \frac{dy}{dx} + 2y = 0 \) with \( y(0) = 1 \), use the Euler's method to obtain \( y(0.1), y(0.2) \) and \( y(0.3) \).