DRR-3249

B. Sc. (Sem. VI) Examination

March / April - 2016

Mathematics : MTH - 601

(Ring Theory)

Time : **2** Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. SC. (SEM. VI)**

Name of the Subject :
☛ **Mathematics : MTH - 601 (Right Theory)**

☛ Subject Code No. : **3 2 4 9** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) First question is compulsory.

(3) Figures to the right indicate marks of corresponding question.

(4) Follow usual notations.

(5) Use of non-programmable scientific calculator is allowed.

1 Answer any FIVE of the following: **10**

(1) Define: Zero divisor with an illustration.

(2) Give an example of a ring having 2016 elements. Is it a field? Why?

(3) If R is a ring, then for all $a, b, \in R$, prove that $a(-b) = -(ab)$.

(4) Which of the rings Z_{18} and Z_{19} are integral domains? Why?

(5) If U is an ideal of a ring R and $1 \in U$ then prove that $U = R$.

(6) Define: Euclidean Domain.

(7) Let R be a Euclidean ring R and $\pi \in R$ is a prime element. If $\pi | ab ; a, b \in R$ then show that $\pi | a \text{ or } \pi | b$.

(8) Find all the associates of $\overline{4}$ in J_6.

2 Answer any TWO of the following: **10**

(1) Prove that any finite integral domain is a field.

(2) If p is a prime number, then prove that J_p, the ring of integers modulo p is a field.

(3) Let R be the set of integers *mod 7* under the addition and multiplication *mod 7* defined as follows :

$\overline{i} + \overline{j} = \overline{k}$; where k is the reminder of $i + j$ on division by 7, and $\overline{i} . \overline{j} = \overline{m}$; where m is the reminder of ij on division by 7.

Prove that R is a commutative ring.

(4) Prove that any field is an integral domain.

3 Answer any TWO of the following: **10**

(1) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.

(2) If $\phi : R \to R'$ is a homomorphism with kernel $I(\phi)$ then prove that $I(\phi)$ is an ideal of R.

(3) If R is a ring with unit element 1 and ϕ is a homomorphism of R onto R', then prove that $\phi(1)$ is the unit element of R'.

(4) Let U be an ideal of R. Let $r(U) = \{x \in R / xu = 0, \textit{for all } u \in U\}$. Prove that $r(U)$ is an ideal of R.

4 Answer any TWO of the following : **10**

(1) Let R be a Euclidean domain and let A be some ideal of R. Then show that there exists an element $a_0 \in A$ such that A consists exactly of all $a_0 x\,; as\ x$ ranges over R.

(2) Let R be a Euclidean ring. Then prove that any two elements $a, b \in R$ always have a greatest common divisor d and $d = \lambda a + \mu b$; for some $\lambda, \mu \in R.$

(3) Explain what do you mean by unit in a commutative ring. Prove that a Euclidean ring always possesses a unit element.

(4) If R is an integral domain with unit element and if $a, b \in R,$ then find the relation between a and b when both $a|b, b|a$ are true.

5 Answer any TWO of the following: **10**

(1) Let R be a Euclidean ring and a_0 is a prime element of R. Prove that $A = (a_0)$ is a maximal ideal of R.

(2) Let R be a Euclidean ring. Prove that every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R.

(3) In a Euclidean ring prove that any two greatest common divisors of two given elements are associates.

(4) Prove that a necessary and sufficient condition that the element a in the Euclidean ring be a unit is that $d(a) = d(1).$
