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DRR-3250

Third Year B. Sc. (Sem. VI) Examination

March / April - 2016

Mathematics : MTH - 602

(Linear Algebra - II)

Time : **2** Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **Third Year B. Sc. (Sem. VI)**

Name of the Subject :
☛ **Mathematics : MTH - 602** *(Linear Algebra - II)*

☛ Subject Code No. : 3 2 5 0 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate the marks of the question.

(4) Follow usual notations.

1 Answer the following as directed: (Any Five) **10**

(1) Check the linearity of the map T : $V_2 \to V_1$ defined by T $(x, y) = x^2 + y^2$.

(2) Let T : $U \to V$ be a linear map. Then prove that $T(-u) = -T(u)$, for every $u \in U$.

(3) Define : Null space and Nullity of a linear map.

(4) Find R(T); for the linear map T : $V_2 \to V_2$ defined by $T(e_1) = (0, 0)$ and $T(e_2) = (0, -1)$.

(5) Prove that the linear map T : $V_2 \to V_2$ defined by $T(e_1) = e_2$ and $T(e_2) = e_1 - e_2$ is one-one.

(6) Let T : $U \to V$ and S : $V \to W$ two linear maps. If ST is onto, then prove that S is onto.

(7) In an inner product space V, prove that, $u \cdot \theta_v = 0$, for every $u \in V$.

(8) Define : Orthogonal Vectors in an inner product space. Give an illustration of orthogonal vectors in the inner product space V_3.

2 Attempt any Two : **10**

(1) Prove that the map T : $V_3 \to V_3$ defined by $T(x_1, x_{2,} x_3) = (x_1, x_2, 0)$ is linear.

(2) Let T : $U \to V$ be a linear map. Prove that R(T) is a subspace of a vector space of V.

(3) Obtain the general rule for a linear map T : $V_2 \to V_2$ such that $T(0,1) = (3,2)$ and $T(3,1) = (2,2)$.

(4) Let T : $U \to V$ be a linear map. Then prove that T is one-one if and only if $N(T) = \{\theta u\}$.

3 Attempt any Two : **10**

(1) Define a non-singular map. Let T : $U \to V$ be a linear map. If $v_1, v_2, \ldots\ldots, v_n$ are LI vectors of R(T) and $u_1, u_2, \ldots\ldots, u_n$ are vectors of U such that $T(u_1) = v_1, T(u_2) = v_2 \ldots, T(u_n) = v_{n,}$ then prove that $u_1, u_2, \ldots\ldots, u_n$ are LI.

(2) Let T : $U \to V$ be a linear map and U be a finite-dimensional vector space. Then prove that $r(T)+n(T)=\dim U$.

(3) Verify the Rank-Nullity Theorem for the linear map T : $V_3 \to V_2$ defined by $T(e_1)=(2,1)$ $T(e_2)=(0,1)$ and $T(e_3)=(1,1)$.

(4) Prove that the linear map T : $V_3 \to V_3$ defined by $T(e_1)=e_1$, $T(e_2)=e_1+e_2$ and $T(e_3)=e_1+e_2+e_3$ is non-singular and find T^{-1}.

4 Attempt any Two : **10**

(1) Let T : $U \to V$ and S : $V \to W$ two linear maps. If S and T are non-singular, then prove that ST is non-singular and $(ST)^{-1}=T^{-1}S^{-1}$.

(2) Determine the matrix $(T:B_1,B_2)$ for the linear map T : $V_3 \to V_2$ defined by $T(x,y,z)=(x+y,y+z)$ relative to the bases : $B_1=\{(1,1,1),(1,0,0),(1,1,0)\}$, $B_2=\{(1,0),(0,1)\}$.

(3) Determine the linear map associated with

the matrix $A=\begin{bmatrix}1 & -1 & 2\\ 3 & 1 & 0\end{bmatrix}$

relative to the bases : $B_1=\{(1,-1,1),(1,2,0),(0,-1,0)\}$,

$B_2=\{(1,1),(2,-1)\}$.

(4) Verify the Rank-Nullity Theorem for the

matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 7 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

5 Attempt any Two : **10**

(1) Define an inner product space. In an inner product space V, prove that :

(i) $(u+v) \cdot w = u.w + v.w$;

(ii) $u.(\alpha v) = \overline{\alpha}(u.v)$; for all $u, v \cdot w \in V$ and for any scalar α.

(2) Define the norm of a vector in an inner product space. In an inner product space establish the Triangle Inequality.

(3) Prove that an orthogonal set of non-zero vectors in an inner product space is LI.

(4) Orthogonalize the LI set $B = \{(0,0,1), (1,1,0), (1,5,2)\}$ of V_3 by the Gram-Schmidt Process.