DRR-3251

B. Sc. (Mathematics) (Sem. VI) Examination

March / April - 2016

MTH-603 : Real Analysis - III

(New Course)

Time : **2** Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. **Fillup strictly the details of ☛ signs on your answer book.**	Seat No. :
Name of the Examination : ☛ **B. Sc. (Mathematics) (Sem. VI)**	
Name of the Subject : ☛ **MTH-603 : Real Analysis - III *(New Course)***	
☛ Subject Code No. : 3 2 5 1 ☛ Section No. (1, 2,.....) : NIL	Student's Signature

(2) All questions carry equal marks and are compulsory.

(3) Follow usual notations.

1 Answer any **five** from the following : **10**

(1) Show that the series $\sum_{n=1}^{\infty}(-1)^n$ is divergent.

(2) Define absolute convergent series and show that $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\dots\dots\dots$ converges absolutely.

(3) Prove that the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is convergent by using Cauchy condensation test.

(4) Give statement of RATIO test.

(5) Prove that the singleton set is of measure zero.

(6) Let $f(x)=x, (0 \le x \le 1)$, let σ be the sub division $\left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ then compute $U[f;\sigma]$.

(7) Define Riemann Integrable function and refinement.

(8) Prove that $\int_a^b(-f)=-\int_a^b f$.

2 Attempt any **two** : **10**

(a) State and prove that the Leibnitz test for the convergence of an alternating series.

(b) If $\sum_{n=1}^{\infty} a_n$ is a convergent series then prove that $\lim_{n\to\infty} a_n = 0$. Is converse true ? Justify your answer.

(c) Does the series $\sum_{n=1}^{\infty} \log\left(1+\frac{1}{n}\right)$ converge or diverge ?

(d) Prove that the series

$$(1-2)+\left(1-2^{\frac{1}{2}}\right)+\left(1-2^{\frac{1}{3}}\right)-\left(1-2^{\frac{1}{4}}\right)+\ldots\ldots\ \text{converges}$$

3 Attempt any **two** : **10**

(a) If $\sum_{n=1}^{\infty} |b_n| < \infty$ and if $\lim_{n\to\infty} \frac{|a_n|}{|b_n|}$ exists, then prove that $\sum_{n=1}^{\infty} |a_n| < \infty$.

(b) If $\{a_n\}_{n=1}^{\infty}$ is non increasing sequence of positive number and if $\sum_{n=1}^{\infty} a_n$ converges then prove that $\lim_{n\to\infty} na_n = 0$.

(c) Using appropriate TEST of Convergence check the convergence for the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

(d) For what values of x does $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converge ?

4 Attempt any **two** : **10**

(a) If each of the subsets $E_1, E_2, E_3, \ldots\ldots\ldots$ of R^2 is of measure zero, then prove that $\bigcup_{n=1}^{\infty} E_n$ is also of measure zero.

(b) If $f \in R[a, b]$ and λ is any real number, then prove that $\lambda f \in R[a, b]$ and $\int_a^b \lambda f = \lambda \int_a^b f$.

(c) Prove that continuous function on the closed bounded interval $[a, b]$ is Riemann lntegrable.

(d) If A is not a measure of zero, if $B \subset A$, and if B is of measure zero then prove that $A - B$ is not a measure of zero. Use it to prove that the set of all irrational numbers is not of measure zero.

5 Attempt any **two** : **10**

(a) If f is a continuous function on the closed bounded interval $[a, b]$, and if $\phi'(x) = f(x)(a \le x \le b)$, then prove that $\int_a^b f(x)dx = \phi(b) - \phi(a)$.

(b) If $f \in R[a, b]$, then prove that $|f| \in R[a,b]$ and $\left|\int_a^b f\right| = \int_a^b |f|$.

(c) If f is a continuous $[a, b]$, then prove that there exists $c \in [a, b]$ such that $\int_a^b f(x)dx = f(c)(b-a)$.

(d) In usual notations prove that $\frac{1}{3\sqrt{2}} \le \int_0^1 \frac{x^2}{\sqrt{1+x}} dx \le \frac{1}{3}$.
