DRR-3252

DRR-3252

B. Sc. (Sem. - VI) Examination

March / April - 2016

Mathematics : Paper - MTH - 604

(Real Analysis - IV)

Time : **2** Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. SC. (SEM. - VI)**

Name of the Subject :
☛ **Mathematics : Paper - MTH - 604**

☛ Subject Code No. : **3 2 5 2** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) First question is compulsory.

(3) Figures to the right indicate marks of corresponding question.

(4) Follow usual notations.

(5) Use of non-programmable scientific calculator is allowed.

1 Answer any FIVE of the following: **10**

(1) Define closed set E and in usual notations prove that $E \subset \overline{E}$.

(2) Prove that any finite subset of a metric space M is closed.

(3) Define :

(i) Bounded set in a metric space M

(ii) Diameter of a set.

(4) Prove that the interval $[1,6]$ is not a connected subset of R_d.

(5) Define :

(i) Cauchy sequence in a metric space M

(ii) Complete metric space.

(6) State : "Nested interval theorem".

(7) Define :

(i) Open covering of a set

(ii) Heine Borel property.

(8) Prove that the closed subset of a compact metric space is compact.

2 Answer any TWO of the following : **10**

(1) Prove the following :

(i) Let G be an open subset of the metric space M. Then prove that $G' = M - G$ is closed.

(ii) Let F be a closed subset of the metric space M. Then prove that $F' = M - F$ is open.

(2) If E is closed subset of a metric space M, then prove that $\overline{E}$ is closed.

(3) Let f be a continuous real-valued function on the metric space M.

Let A be the set of all $x \in M$ such that $f(x) \geq 0$. Then prove that A is closed.

(4) If $\mathcal{F}$ is any family of closed subsets of a metric space M, then prove that $\bigcap_{F \in \mathcal{F}} F$ is also closed.

3 Answer any TWO of the following : **10**

(1) Prove that the range of a continuous function defined on a connected metric space is also connected.

(2) If a subset A of the metric space $\langle M, \rho \rangle$ is totally bounded then prove that A is bounded.

(3) If A is a connected subset of the metric space M and if $A \subset B \subset \overline{A}$, then prove that B is connected.

(4) If A_1 and A_2 are connected subsets of a metric space M, and if $A_1 \cap A_2 \neq \varnothing$, then prove that $A_1 \cup A_2$ is also connected.

4 Answer any TWO of the following : **10**

(1) If $\langle M, \rho \rangle$ is a complete metric space and A is closed subset of M then prove that $\langle A, \rho \rangle$ is also complete.

(2) Prove that R^2 is complete.

(3) Prove that the interval $(0,1)$ with absolute value metric is not a complete metric space, but it is complete with the metric of R_d.

(4) Give statement of "Picard's fixed point theorem". If $f(x) = x^2; 0 \leq x \leq \frac{1}{3}$, then prove that T is a contraction on $\left[0, \frac{1}{3}\right]$.

5 Answer any TWO of the following : **10**

(1) Prove that the metric space $\langle M, \rho \rangle$ is compact if and only if whenever $\mathcal{F}$ is a family of closed subsets of M with Finite Intersection Property, then $\bigcap_{F \in \mathcal{F}} F \neq \emptyset$.

(2) Prove that the metric space $\langle M, \rho \rangle$ is compact if and only if every sequence of points in M has a subsequence converging to a point in M.

(3) If A and B are compact subsets of R^1, then prove that $A \times B$ is a compact subset of R^2.

(4) Prove that a connected subset of R_d is compact. Also give an illustration of a connected subset of R^1 that is not compact.
