* D R R - 3 2 5 4 *

DRR-3254

B. Sc. (Sem. VI) Examination

March / April - 2016

Mathematics : Paper - MTH-606

(Number Theory - II)

Time : Hours] [Total Marks : **50**

Instruction :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. SC. (SEM. VI)**

Name of the Subject :

☛ **MATHEMATICS : PAPER-MTH-606 (NUMBER THEORY - II)**

☛ Subject Code No. : **3 2 5 4** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) First question is compulsory.

(3) Figures to the right indicate marks of corresponding question.

(4) Follow usual notations.

(5) Use of non-programmable scientific calculator is allowed.

1 Answer any FIVE of the following : 10

(1) Write the condition that the linear congruence $ax \equiv b \pmod{n}$ has a solution.

(2) Define: Pseudoprime.

(3) Find the value of $\tau(360)$ and $\sigma(360)$.

(4) For multiplicative function *f* if $f(n) \neq 0$ then prove that $f(1) = 1$.

(5) Obtain the highest power of 7 which divide 100! .

(6) Find the value of $\sum_{k=1}^{25} \mu(k!)$.

(7) For any integer n prove that $[x+n] = [x]+n$.

(8) If n is an odd integer then prove that $\phi(2n) = \phi(n)$.

2 Answer any **TWO** of the following: **10**

(1) State and prove Chinese Remainder Theorem.

(2) Solve the linear congruence $6x \equiv 15 \ (mod\ 21)$.

(3) Solve the following simultaneous congruences:

$x \equiv 1 \pmod 3,\ x \equiv 2 \pmod 5,\ x \equiv 3 \pmod 7$.

(4) Obtain three consecutive integers such that $5^2 \mid a,\ 3^2 \mid a+1$ and $2^2 \mid a+2$.

3 Answer any **TWO** of the following: **10**

(1) State and prove Wilson's Theorem.

(2) If p and q are distinct primes such that $a^p \equiv a \pmod q$ and $a^q \equiv a \pmod p$ then prove that $a^{pq} \equiv a \pmod{pq}$.

(3) For a prime p, prove that $1^{p-1} + 2^{p-1} + \ldots + (p-1)^{p-1} \equiv -1 \pmod p$.

(4) Verify that $4(29!) + 5!$ is divisible by 31.

4 Answer any **TWO** of the following: **10**

(1) For any positive integer n prove that

$$\sum_{d|n} \mu(d) = \begin{cases} 1; if\ n = 1 \\ 0; if\ n > 1 \end{cases}.$$

(2) If F is a multiplicative function and $F(n) = \sum_{d|n} f(d)$ then prove that f is also multiplicative function.

(3) Find the number of zeros in which 1000! terminates.

(4) For any positive integer n, show that $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$.

5 Answer any **TWO** of the following: **10**

(1) State and prove Euler's Theorem.

(2) For $n > 2$, prove that $\phi(n)$ is an even integer.

(3) Find the last two digits in the decimal expansion of 3^{253}.

(4) If n is an odd integer such that $5 \mid n$ then show that n divides the integer whose all digits are 1.
