DRR-3258

Third Year B. Sc. (Sem. - VI) Examination

March / April - 2016

Fourier Transform : 6004 (3)

(Generic Elective)

Time : Hours] [Total Marks : **50**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **Third Year B. Sc. (Sem. - VI)**

Name of the Subject :
☛ **Fourier Transform : 6004 (3)**

☛ Subject Code No. : **3 2 5 8** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding question.

1 Answer the following: (Any Five) **5**

(1) What is the one dimensional heat-flow equation?

(2) Write down the parseval's identities for Fourier cosine transform.

(3) Define convolution of two functions.

(4) Define Integral equation.

(5) State the equation of Fourier sine and cosine transforms.

(6) Define Fourier transform.

(7) Write down one dimensional wave equation for vibrating string.

(8) Prove that $F\left[\frac{\partial^2 u}{\partial x^2}\right] = -s^2 F[u]$.

2 (a) Derive Fourier sine and cosine Integral. **8**

OR

(a) Derive Fourier Integral Theory of function f(x).

2 (b) Attempt any one. **7**

(1) Find the Fourier Transform of $f(x)=\begin{cases} 1; |x|<1 \\ 0; |x|>1 \end{cases}$

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx.$

(2) Express the function $f(x)=\begin{cases} 1; 0 \le x \le \pi \\ 0; \quad x>\pi \end{cases}$ as a Fourier

Sine Integral. Hence evaluate $\int_0^\infty \frac{1-\cos \pi\lambda}{\lambda} \sin x\lambda d\lambda$

3 (a) State parseval's identity for Fourier Transform and prove it. **8**

OR

(a) Derive Fourier transform of the derivatives of a function.

3 (b) Attempt any one. **7**

(1) If $f(x)=e^{-ax}$ & $g(x)=\begin{cases} 1; 0<x<1 \\ 0; \quad x>a \end{cases}$ Show that

$\int_0^\infty \frac{\sin at}{t\left(a^2+t^2\right)} dt = \frac{\pi}{2}\frac{1-e^{-a^2}}{a^2}$, by Parseval's identity.

(2) Using parseval's identity, show that. $\int_0^\infty \frac{x^2 dx}{\left(x^2+1\right)^2}=\frac{\pi}{4}$,

where $f(x)=e^{-x}$.

4 (a) If the initial temperature of an infinite bar is given by $\theta(x) = \begin{cases} \theta_0; |x| < a \\ 0; |x| > a \end{cases}$, determine the temperature at any point x and at any instant t. **8**

OR

(a) Derive Fourier transform of the derivatives of a function.

4 (b) Attempt any one. **7**

(1) Evaluate $L^{-1}\left\{\frac{4s+5}{(s+2)(s-1)^2}\right\}$ by method of residues

(2) Using the method of residues, evaluate

$$L^{-1}\left\{\frac{1}{(s+1)(s-1)^2}\right\}$$