DE-1322
M. Sc. (Sem. I) (Electronics) Examination
March / April - 2016
EL - 411 : Mathematical Methods

Time : 3 Hours] [Total Marks : 70

Instructions :

1. Attempt any two questions out of the (a), (b) and (c) below.
   (a) What do you mean by separable and exact differential equations? State necessary and sufficient condition for an exact differential equation. Derive the integrating factor for general linear first order equation and solve \( xy' + (1 + xy) = e^x \).
   
   (b) (i) Using WKB method solve \( y'' + xy = 0 \) for \( x >> 0 \).
   (ii) Find the general solution of \( y'' + 3y' + 2y = x^3 + x \) using method of undetermined coefficients.
   
   (c) (i) What does element \([2 4 1 5 3]\) mean? Prove that \([2 4 1 5 3] [5 1 2 3 4] = [1 3 5 4 2]\).
   (ii) Define: Symmetric Group, Subgroup and Cosets

2. Attempt any two questions out of the (a), (b) and (c) below.
   (a) Explain the limitations of Fourier Transform and role of Heaviside step function in Laplace Transforms. Obtain the Laplace transform and Laplace inverse transform. Also find out the Laplace transform of unity.
   
   (b) (i) Prove that \( \mathcal{F}[f''(x)] = -\alpha^2 \mathcal{F}[f(x)] \).
   (ii) Find \( \mathcal{L}[\sin \alpha x] \).
   
   (c) (i) Solve the differential equations of coupled pendulums using Laplace transform with initial conditions \( x_1 = x_2 = 0, x_1' = v \) and \( x_2' = 0 \) at \( t = 0 \).
   (ii) Find \( \mathcal{L}[xf(x)] \).

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3 Attempt any two questions out of the (a), (b) and (c) below.

(a) Derive the generating function, Schlafli’s integral representation and Bessel’s integral formula for the Bessel’s function.

(b)(i) Deduce the recursion relation: \( P'_n(z) - 2zP'_{n-1}(z) + P'_{n-2}(z) = P_{n+1}(z) \)

(ii) Prove that \( J_{m-1}(x) - J_{m+1}(x) = 2J_m(x) \).

(c)(i) Find the Bessel functions \( J_{3,2}(x) \).

(ii) Evaluate spherical harmonics for \( l = 2 \).

4 Attempt any two questions out of the (a), (b) and (c) below.

(a) Define: Residue. State and prove Residues theorem. Discuss the different methods to find residue.

(b)(i) Prove that \( \int_0^{2\pi} \cos 2\theta \frac{d\theta}{(5 + 4\cos \theta)} = \frac{\pi}{6} \) using residues theorem.

(ii) Find the residues of \( \frac{z^2 - z^3 + 1}{z^3} \) at infinity.

(c)(i) Using contour method, prove that \( \int_{-\infty}^{\infty} \frac{x^2 + x + 2}{x^4 + 10x^2 + 9} \, dx = \frac{5\pi}{12} \)

(ii) Discuss the types of boundary conditions appropriate for the hyperbolic, elliptic and parabolic equations.

5 Attempt any two questions out of the (a), (b) and (c) below.

(a) Explain term: Roots of an equation. Deduce the Newton Raphson and Regula Falsi methods to find out the root of an equation.

(b)(i) Compute the real root of \( x \log_{10} x - 1.2 = 0 \) correct to five decimal places using Regula Falsi method.

(ii) Write a note on: Poisson distribution

(c)(i) Using Lagrange’s interpolation formula, find the form of function \( f(x) \) from the following data.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>3</td>
<td>12</td>
<td>15</td>
<td>-21</td>
</tr>
</tbody>
</table>

(ii) Find a root to three significant figures of the equation: \( x^3 + 10 \, x = 4 \) using Newton Rapson method.