DE-1331
M. Sc. (Sem. I) (Mathematics) Examination
March / April - 2016
Fourier Analysis

Time : 3 Hours] [Total Marks : 70

Instructions :
(1) Fill up strictly the details of signs on your answer book.

Name of the Examination :
M. SC. (SEM. I) (MATHEMATICS)

Name of the Subject :
FOURIER ANALYSIS

(2) Answer all questions.
(3) Follow usual notation.
(4) Figures to right indicate marks of the questions.

1. Attempt any two [14]
   1) Derive the complex Fourier series for the interval \([c, c+2l]\).
   2) Expand the function \(f(x) = 2 - x\), \(0 < x < 1\) in terms of half range cosine series.
   3) If the Fourier series of \(f(x)\) is defined over the interval \([c, c+2l]\), then derive the Parseval's identity for the given Fourier series.

2. Attempt any two [14]
   1) Expand the \(2\pi\) periodic function \(f(x) = 2x - x^2\), \(-1 < x < 1\) in terms of Fourier series.
   2) Find the complex form of Fourier series for the function \(f(x) = \cosh ax\), \(x \in [-\pi, \pi]\)
   3) Prove:
      \[ \frac{1}{n} \int_{-\pi}^{\pi} D_n(u) du = 1 \quad \text{where } D_n(u) \text{ is Dinchlet's kernel} \]
      \[ D_n(u) = \frac{\sin(n+\frac{1}{2})u}{2 \sin\left(\frac{u}{2}\right)} \]

3. Attempt any two [14]
   1) Find the fourier transform of the function \(f(x) = \begin{cases} 1 & |x| < 1 \\ 0 & |x| > 1 \end{cases}\) and hence evaluate the following
      (a) \( \int_{-\infty}^{\infty} \cos xt \sin t \ dt \)
      (b) \( \int_{-\infty}^{\infty} \sin t \ dt \)
   2) Define convolution of two functions. Prove that if \(F(f(x)) = F(k)\) and \(F(g(x)) = G(k)\) then \(F(f(x) \ast g(x)) = F(k) \cdot G(k)\)

DE-1331] [Contd...
3) Show that

(a) \[ \mathcal{F}_c(e^{-ax}) = \frac{2}{\pi} \left( \frac{-a}{a^2 + k^2} \right); \ a > 0 \]

(b) \[ \mathcal{F}_c(e^{-ax}) = \frac{2}{\pi} \left( \frac{k}{a^2 + k^2} \right); \ a > 0. \]

4 Attempt any two \[ [14] \]

1) Define Complete orthonormal system. Show that The expansion co-efficient of L^2-integrable function converges to zero as n is increased indefinitely for an orthogonal system

2) Find the cosine transform for \( f(x) = \begin{cases} \cos x; & 0 < x < \pi \\ 0; & x > \pi \end{cases} \)

3) Prove that if \( f(x) \) be an absolutely integrable function of period \( 2\pi \) then, at every continuity point where the right and left hand derivatives exists, then the Fourier series of \( f \) converges to the value of \( f(x) \).

5 Attempt any two \[ [14] \]

1) If function \( f(x) \) satisfied Dirichlet’s condition then derive the Fourier sine integral formula.

2) Solve the Heat equation \( u_t = u_{xx} \) subject to the following condition

\( \begin{align*} 
& (a) \quad u_x(0,t) = 0 \\
& (b) \quad u(x,0) = \begin{cases} x; & 0 \leq x < 1 \\
& 0; & x \geq 1 \end{cases} \\
& (c) \quad u(x,t) \text{ is bounded} 
\end{align*} \)

4) Define Mean-Square error. Prove that all the polynomials of the \( n \)th order with respect to an orthonormal system \( \{ \varphi_n(x) \} \), the best approximation for \( f(x) \in L^2 \) is given by the \( n \)th partial sum of its Fourier series with respect to this system.