M. Sc. (Sem. III) (Mathematics) Examination
March / April - 2016
602 : Advanced Functional Analysis

Time : 3 Hours] [Total Marks : 70

Instructions :

(1) Fill up strictly the details of signs on your answer book.

M. SC. (SEM. 3) (MATHEMATICS)

Name of the Subject :

602 : ADVANCED FUNCTIONAL ANAL.

Seat No.:

(2) Attempt all questions.

(3) Figures to the right indicate full marks.

(4) Follow usual notations and conventions

Q-1 Attempt any TWO.

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(1) State and prove Reisz’s representation theorem.

(2) Let M be a subset of an inner product space X then prove that
   a) If M is total in X then there does not exist a non-zero \( x \in X \) which is
      orthogonal to every elements of X, i.e. \( x \perp M \Rightarrow x = 0 \)
   b) If X is complete, the condition is sufficient for totality of M in X.

(3) Let H be a Hilbert space then prove that
   a) If H is separable, every orthogonal set in H is countable.
   b) If H contains an orthogonal sequence which is total in H then H is
      separable.

Q-2 Attempt any TWO.

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(1) Let \( T : H \to H \) be bounded linear operator on Hilbert space H then prove
   a) If T is self adjoint then \( (Tx,x) \) is real.
   b) If H is complex and \( (Tx,x) \) is real, \( \forall x \in H \) then \( T : H \to H \) is self adjoint.

(2) Prove that a linear operator T on a Hilbert space H is unitary iff Hilbert
    adjoint of T exist and \( T^*T = TT^* = I \)
(3) Define PO-set and Chain. Prove that every vector space $X \neq \{0\}$ has Hammel basis.

**Q-3 Attempt any TWO.**

(1) Prove that the adjoint operator $T^\times$ of a bounded linear operator $T : X \to Y$ is bounded, linear and $\|T^\times\| = \|T\|$ where $T^\times : Y' \to X'$ is defined as $(T^\times g)(x) = g(Tx) = f(x), \forall x \in X, g \in Y'$ and $f \in X'$.

(2) Show that Uniformly operator convergence $\not\equiv$ Strongly operator convergence $\not\equiv$ Weakly operator convergence.

(3) Prove that a canonical mapping $C$ given by $x \mapsto g_x$ is an isomorphism of a normed space $X$ into a normed space $\mathcal{R}(C)$.

**Q-4 Attempt any TWO.**

(1) Derive the relation between the adjoint operator and Hilbert adjoint operators.

(2) State and prove Uniform boundedness theorem.

(3) Let $X$ and $Y$ be two Banach space and $T_n \in B(X, Y)$ then prove that a sequence $(T_n)$ is strongly operator convergence iff
   a) Sequence $(\|T_n\|)$ is bounded.
   b) Sequence $(T_n x)$ is Cauchy in $Y$ for every $x$ in a total subset $M$ of $X$.

**Q-5 Attempt any TWO.**

(1) Show that if $X$ be a normed linear space and $x_0(\neq 0) \in X$ be arbitrary then there exists a bounded linear functional $\tilde{f}$ on $X$ such that $\|\tilde{f}\| = 1$ and $\tilde{f}(x_0) = \|x_0\|$.

(2) Prove that if $X$ be the set of all polynomials and norm is defined on $X$ as $\|x\| = \max_j |a_j|$, then $X$ is not complete.

(3) Prove that every Hilbert space $H$ is reflexive.