M. Sc. (Sem. III) Examination
March / April – 2016
Mathematics : Paper - 603
(Linear Algebra)

Time : Hours] [Total Marks : 70

Instructions :

(1) Fill up strictly the details of signs on your answer book.

Name of the Examination :
M. SC. (SEM. III)

Name of the Subject :
MATHEMATICS : PAPER - 603

Seat No. : Section No. (1, 2, ....)

(2) All questions are compulsory.
(3) Figures to the right indicate full marks.
(4) Follow usual notations and conventions.
(5) This paper contains five questions.
(6) Each question is of 14 Marks.

1 Answer any TWO

(a) Define: (i) linear Operator (ii) Kernel (iii) Range of the matrix. (iv) Nonsingular (v) Basis.

(b) State and prove Rank – Nullity Theorem.

(c) Composition Operator. Also Define , J : f_n → f_{n+1} and , D : f_n → f_{n+1} by, D(f(x)) = f'(x) and J(f(x)) = \int_0^x f(t)dt then prove that D is left inverse of J and J is right inverse of D.

2 Attempt any TWO

(a) Let T be the matrix operator T(x) = Ax with A given as
\[
\begin{bmatrix}
1 & 0 & 2 & 3 & 5 \\
2 & 0 & -2 & 1 & 6 \\
3 & 0 & 2 & 1 & 3
\end{bmatrix}
\]
Find a basis for N (T) and find its dimension. Find r (T).
(b) Define: matrix operator. Also prove that a linear operator is completely
determined by its values of basis.

(c) Let \( T: p_2 \rightarrow p_2 \) defined by \( T(p(x)) = \frac{1}{2} \int_{-1}^{1} (15t^2 + 3xt + 6x^2)p(t)dt. \)
Construct \( T^{-1} \) and find \( T^{-1}(t^2). \)

3 Answer any TWO

(a) Define \( T: p_2 \rightarrow p_2 \) by,
\[
T_p(x) = (1 + 2x^2)p''(x) + (1 - 2x)p'(x) + p(x) + xp(0)
\]
then find a
matrix of \( T \) with respect to basis \( \{1, 1 + x, 1 + x + x^2\} \), also check that \( T \)
is invertible or not.

(b) Define: (i) Norm Space (ii) Inner product space (iii) Orthogonal Vector
(iv) Fourier co-efficient (v) Direct Sum.

(c) Let \( v_i(t) = t^i \); \( i = 0, 1, 2, 3 \). And let \( \langle x, y \rangle = \int_0^\infty x(t)y(t)\ e^{-t}dt \) be
given inner product. Find orthogonal polynomials, \( e_i \); \( i = 0, 1, 2, 3 \). with
respect to given inner product.

4 Attempt any TWO

(a) Let \( A \) be an \( m \times n \) matrix with enries in \( F \) and linearly independent
columns then prove that there is an \( m \times n \) matrix \( Q \), and \( n \times n \) matrix \( R \),
both having entries in \( F \) such that (i) \( A = QR \) (ii) \( Q^HQ = I_n \) (iii) \( Q \) and \( R \)
are unique when \( m = n \); \( Q \) is unitary.

(b) Let \( M \) be a finite dimensional subspace of \( V \) then prove that projection
of \( f \) on \( M \) exists and is unique.

(c) State and prove the method of computation of a projection using a
spanning set.

5 Answer any TWO

(a) Let \( M \) be a finite dimensional subspace of \( V \). Also \( V \) is finite
dimensional vector space. Then prove that :

\( a)V = M \oplus M^\perp \)
\( b)(M^\perp)^\perp = M. \)
\( c) If \ V \ is finite dimensional then show that Dim \ V = \ dim M + \ dim M^\perp. \)
(b) Find the orthogonal projection of $|t|$ on $p_2$, using the real inner product
$$\langle f, g \rangle = \int_{-1}^{1} f(t) g(t) dt.$$ 

(c) Define: Projection Operator and Approximation Problem. Also find the projection operator and its matrix in the standard basis with the help of orthogonal basis $\{e_1, e_2\} = \{(1, -1, 0)^T, (1, 1, -2)^T\}$