DF-1488
M. Sc. (Sem. III) Examination
March / April - 2016
Mathematics
(6007 : Integral Transform - I)

Time : 3 Hours] [Total Marks : 70

Instructions :

(1) Fill up strictly the details of [ signs on your answer book.
Name of the Examination :
M. SC. (SEM. III)
Name of the Subject :
MATHEMATICS
Subject Code No. : 1488 [Section No. (1, 2,...) ]

(2) All questions are compulsory.
(3) Figures to the right indicate full marks.
(4) Follow usual notations and conventions.
(5) This paper contains five questions.
(6) Each question is on 14 marks.

1 Answer any TWO

(a) Define Laplace Transform.
   State and prove existence theorem for Laplace Transform.
(b) (i) State and prove Change of Scale property for Laplace transform.
   (ii) If \( \mathcal{L}[f(t)]=\mathcal{F}(s) \) then prove that \( \mathcal{L}\left[\int_0^t f(y)dy\right]=\frac{1}{s}\mathcal{F}(s) \)
(c) (i) In usual notations prove that: \( \mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\mathcal{F}(s) \)
   (ii) If \( f(t)=t^2 ; 0<t<2 \) and \( f(t+2)=f(t) \) then find \( \mathcal{L}[f(t)] \)

2 Answer any TWO

(a) Define exponential integral function and derive it's Laplace transform
(b) If \( \mathcal{L}[f(t)]=\mathcal{F}(s) \) then prove that \( \mathcal{L}\left[\frac{d}{dt} f(t)\right]=\int_s^\infty f(x)dx \) Provided \( \lim_{t\to0^+} \frac{1}{t} f(t) \) exist.
(c) Prove that \( \mathcal{L}\left[\frac{\sin(t)}{t}\right]=\tan^{-1}\left[\frac{1}{s}\right] \) and hence find \( \mathcal{L}\left[\frac{\sin(at)}{t}\right] \).
   Does Laplace transform of \( \frac{\cos(at)}{t} \) exist?

DF-1488] 1 [Contd...
3 Answer any TWO

(a) Prove that:  \[ L^{-1} \left[ \frac{f(s)}{s^2} \right] = \int_0^\infty f(u) \, du \, dv \]

(b) State and prove convolution theorem for Laplace Transform and

hence Evaluate:  \[ L^{-1} \left[ \frac{s^2}{(s^2 + 4)^2} \right] \]

(c) State Heaviside expansion formula and using this formula evaluate

\[ L^{-1} \left[ \frac{s^2 - 6}{s^3 + 4s^2 + 3s} \right] \]

4 Solve any TWO

(a) Using Laplace transform, prove that:  \[ \int_0^\infty \sin(x^2) \, dx = \frac{1}{2} \sqrt{\frac{\pi}{2}} \]

(b) Using Laplace transform, prove that:  \[ \int_0^\infty e^{-x^2} \, dx = \frac{1}{2} \sqrt{\pi} \]

(c) Using Laplace transform, prove that:

(i)  \[ \int_0^\infty e^{-3t} \sin(t) \, dt = \frac{3}{50} \]

(ii)  \[ \int_0^\infty \sin^2(t) \, dt = \frac{\pi}{2} \]

5 Solve any TWO

(a) Using Laplace transform solve the boundary value problem:

\[ (D + D^2) y(t) = 2 \; \text{where} \; y(0) = 3 \; \text{and} \; y'(0) = 1 \]

(b) Using Laplace transform solve the integral equation:

\[ f(t) = t + 2 \int_0^t \cos(t-u) f(u) \, du \]

(c) Using Laplace transform solve the differential equation:  \[ ty''(t) + y'(t) + ty(t) = 0 \]

Under the conditions that \( y(0) = 1 \) and \( y(t) \) and its derivative have transforms.