DMM-1590
M. Sc. (Mathematics) (Sem. IV) Examination
April/May - 2016
Paper - 703 : Ad. Topology

Time : 3 Hours] [Total Marks : 70

Instructions :
(1) Fill up strictly the details of signs on your answer book.
Name of the Examination : M. Sc. (Mathematics) (Sem. IV)
Name of the Subject : Paper - 703 : Ad. Topology

(2) All questions are compulsory.
(3) Notations used are standard.
(4) Figure at the right end of the first line of each question indicates full marks.
(5) Each question is of 14 marks.

1 Attempt any two :
   (a) Define $T_1$-space. Prove that a topological space is a $T_1$-space if and only if each point is a closed set. 7
   (b) Define Hausdorff space. Prove that every compact subspace of a Hausdorff space is closed. 7
   (c) Prove that every compact Hausdorff space is normal. 7

2 Solve any two :
   (a) Prove that the product of any non empty class of Hausdorff space is a Hausdorff space. 7
   (b) Prove that a one-to-one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism. 7
   (c) Let $A$ and $B$ be disjoint closed subspaces of a normal space $X$. Prove that there exists a continuous real function $f$ defined on $X$, all of whose values lie in the closed unit interval $[0, 1]$ such that $f(A) = 0$ and $f(B) = 1$. 7
Answer any two:

(a) What is connected space? Prove that any continuous image of a connected space is connected.

(b) Prove that the spaces $\mathbb{R}^n$ and $\mathbb{C}^n$ are connected.

(c) Prove that the product of any non-empty class of connected space is a connected.

Attempt any two:

(a) What do you understand by disconnected space? Prove that a topological space $X$ is disconnected if and only if there exists a continuous mapping of $X$ onto the discrete two point space $\{0, 1\}$.

(b) Prove that the components of a totally disconnected space are its points.

(c) If a compact Hausdorff space $X$ is totally disconnected then prove that $X$ has an open base whose sets are closed in $X$.

Attempt any two:

(a) Let $A$ be connected sub space of a topological space $X$ and $B$ be a subspace of $X$ and such that $A \subseteq B \subseteq \overline{A}$ then prove that $B$ is connected.

(b) For an arbitrary topological space $X$ prove that each connected subspace of $X$ is contained in a component of $X$. Also prove that each component of $X$ is closed.

(c) If $A_1, A_2, \ldots, A_n, \ldots$ is a sequence of connected subspaces such that each intersects its successor, then show that $\bigcup_{n=1}^{\infty} A_n$ is connected.