DPP-1393

M. Sc. (Mathematics) (Sem. II) Examination
April / May - 2016
504 - Discrete Structure

Time : Hours] [Total Marks : 70

Instructions :
(1) Fill up strictly the details of signs on your answer book.

(2) There are five questions in this question paper.
(3) Answer all questions.
(4) Figures to the right indicates mark of the question.

1 Define any SEVEN. 14

1. Alphabet
2. Antiatom
3. Direct product of lattices
4. Substitution property
5. Context free grammar
6. Prime implicant cube
7. Free Boolean algebra
8. Adjacent minterms
9. Metalanguage
10. Length of a formula

2 (a) For the set of natural numbers $N$, prove that $< N, +>$ is a semigroup. Is the set odd non-negative integers form a subsemigroup for $< N, +>$? Justify your answer.

OR

(a) Let $S = \{a, b, c\}$ then prove that $< \rho(S), \cup>$ and $< \rho(S), \cap>$ are monoids.

DPP-1393] 1 [Contd...
(b) Attempt any THREE.

1. Prove that for a semigroup homomorphism commutativity is preserved.
2. Prove that the quotient algebra corresponding to a semigroup is also a semigroup.
3. What are the disadvantages of residue number system?
4. Prove that concatenation is not commutative except when $V$ contains a single element.

3 (a) Discuss the application of Euler's theorem to find $x$.

OR

(a) Prove that if $a$ and $m$ are relatively prime integers then $a' = a^{f(m)-1} \mod m$.

(b) Attempt any THREE.

1. Write a note on number systems.
2. Prove that the composite mapping of two semigroup homomorphism is again a semigroup homomorphism.
3. Let $f$ be a homomorphism from $< X, \circ >$ onto $< Y, \oplus >$, $E$ be a congruence relation corresponding to $f$ and $g_E$ be the natural homomorphism from $< X, \circ >$ to $< \frac{X}{E}, \ast >$, then prove that $h: \frac{X}{E} \to Y$ is a isomorphism where $h[x] = f(x); x \in X$.
4. Find '$x$' for $< 2, 3, 7 > \in Z_{42}^*$ in the mixed based number system.

4 (a) Define partial ordering relation and give an example with justification.

OR

(a) Prove that the operations of meet and join on a lattice is commutative, associative, idempotent and satisfies the law of absorption.
(b) Attempt any THREE.

1. Prove that \(< S_{30}, D >\) is a lattice.
2. Prove that 0 is the only complement of 1 in a lattice.
3. Prove that in a Boolean algebra, complement of every element is unique.
4. Prove that \(\bigoplus_{i=0}^{2^n-1} min_i = 1\) for three variables.

5 (a) Prove that for any mapping from a Boolean algebra which preserves the operation \(*\) and \(\oplus\), the image set is also a Boolean algebra.

OR

(a) Represent the Boolean function \(f(a, b, c, d) = \Sigma(3, 6, 9, 12)\) using the following methods:
(i) Circuit diagram representation
(ii) Tabular representation
(iii) n-space representation.

(b) Attempt any THREE.

1. Use the Quine–Mcclusky algorithm to minimize the Boolean function \(f(a, b, c, d) = \Sigma(0, 2, 3, 5, 8, 12, 14)\).
2. Write a note on “switching algebra”.
3. Any antiatom which is \(\geq a\), then prove that it must appear in the meet representation of the element \(a\).
4. Obtain the sum of product and product of sum canonical form in four variables for the Boolean expression \(x_1 \ast x_2\).