DPP-1394
M. Sc. (Sem. II) (Mathematics) Examination
April/May – 2016
Paper - 505 : Numerical Analysis

Time : 3 Hours] [Total Marks : 70

Instructions :

(1) Fill up strictly the details of signs on your answer book.

(2) Attempt all questions.

(3) Figures to the right indicates marks.

(4) Follow the usual notations and conventions.

Q-1 Attempt any two [14]

(1) Use the Iteration method to obtain a real root, correct to three decimal places, for the equation $e^{-x} = 10$

(2) Obtain a root, correct to three decimal places, for the equation $x^3 - 18 = 0$ using Bisection method

(3) Use the Newton Raphson method to obtain a root, correct to three decimal places, for the equation $x \sin x + \cos x = 0$

Q-2 Attempt any two [14]

(1) Compute three steps of the Aitken’s $\Delta^2$ method for the equation $x^3 - 6x^2 + 11x - 6 = 0$.

(2) Use the synthetic division and perform three iterations of the Birge-Vieta method to find the positive root of the polynomial $P_3(x) = 2x^3 - 5x + 1 = 0$. Use initial approximation $P_0 = 0.5$

(3) Derive Newton-Raphson multiple root method.

DPP-1394] 1 [Contd...
Q-3 Attempt any two

(1) Solve system of equations by LU decomposition method with partial pivoting.

\[
\begin{align*}
2x + 3y + z &= 9 \\
x + 2y + 3z &= 6 \\
3x + y + 2z &= 8
\end{align*}
\]

(2) Find inverse of the matrix using Gauss-Jordan method for the matrix

\[
A = \begin{bmatrix}
5 & -2 & 4 \\
-2 & 1 & 1 \\
4 & 1 & 0
\end{bmatrix}
\]

(3) Derive the Chebyshiev Method to find solution of the equation.

Q-4 Attempt any two

(1) Find all eigenvalues and the corresponding eigenvector for the matrix:

\[
A = \begin{bmatrix}
1 & 3 & -1 \\
3 & 2 & 4 \\
-1 & 4 & 10
\end{bmatrix}
\]

(2) Use the Gauss-Seidal iteration method to solve the system of equation to three decimal places. Start with the initial approximation with \((x_0, y_0, z_0) = (2, 3, 0)\)

\[
\begin{align*}
5x - y + z &= 10 \\
2x + 4y &= 12 \\
x + 2y + 5z &= -1
\end{align*}
\]

(3) Find the interpolating polynomial using Lagrange’s interpolation formula for the given data. hence, find value at \(x = 5\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>-12</td>
<td>0</td>
<td>12</td>
<td>24</td>
</tr>
</tbody>
</table>

Q-5 Attempt any two

(1) The following values of the function are given

<table>
<thead>
<tr>
<th>(x)</th>
<th>0.61</th>
<th>0.62</th>
<th>0.63</th>
<th>0.64</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>1.8404</td>
<td>1.8589</td>
<td>1.8776</td>
<td>1.8964</td>
<td>1.9155</td>
</tr>
</tbody>
</table>

Construct the interpolating polynomial that fits the data using suitable Newton’s difference interpolation. Hence find \(f(0.644)\).

(2) Define divided difference operator. Derive Newton’s divided difference method to find nth degree polynomial.

(3) Perform two steps of the Muller’s method to find the root of the equation \(f(x) = x^3 - (1/2)\). Take \(x_0 = 0\), \(x_1 = 1\), \(x_2 = 0.5\)